

**MAPLE LEAF INTERNATIONAL SCHOOL**  
**COURSE SYLLABUS 2018-2019**

# Calculus 12

## Course overview

This course covers single-variable differential and integral calculus. In the first term, students will learn the mathematical language of limits and derivatives and apply them to a multitude of problems in the real world. In the second term, students will discover anti-differentiation and use integral calculus to solve application problems.

## Textbook

The main textbook for this course will be Stewart Calculus 8<sup>th</sup> edition. Another textbook by Larson & Edwards can be used as additional study material. Both of these textbooks are available in the library.

## Calculators

Scientific calculators are permitted in Exams and Unit Tests. Any Scientific Calculator that does not have a CALC [solve] button is acceptable. Graphing calculators are useful tools for learning Calculus, but they are not permitted in Exams and Unit Tests.

Outline of course content	Textbook sections	weight
Unit 1	Chapter 1	5 %
Unit 2	Chapter 2, 3.2, 3.4	8 %
Unit 3	3.5, 3.1, 3.6, 3.3, 4.1, 4.3, 4.5	8 %
Unit 4	3.9, 4.7, 3.4, 4.8, 4.2, 4.4	8 %
Midterm Exam		15 %
Unit 5	Chapter 5, 7.7, 6.1, 6.5	8 %
Unit 6	9.1, 9.2, 9.3, 3.8	8 %
Unit 7	7.3, 7.2	5 %
Unit 8	7.1, 7.4, 7.8, 6.2	5 %
Final Exam		30 %

## Assessments

Within each unit students will attempt a small number of quizzes and projects and write a unit test to demonstrate their understanding of course content. Midterm exams will be done during Midterm Assessment Week. It will be a balanced assessment designed to test your understanding of the units covered in units 1 to 4. The Final Exam will be written at the end of the semester. Final Exam period. It will cover all of the topics taught in the course and is worth a significant portion of your overall grade.

## Reassessment

Mathematics is a cumulative subject with each unit building upon previous knowledge. In order to have a strong understanding of mathematics, one must be well versed in all units. The mathematics department understands that students need time to see the big picture and that some students may not completely understand it until they're studying for the final exam. Therefore, for students who achieve a final exam mark that is higher than their pre-exam mark, the final exam will be worth 50% of the course instead of 30% in order to give a more accurate indication of the student's up-to-date level of understanding.

## Letter Grade Breakdown

A	86% - 100%
B	73% - 85%
C+	67% - 72%
C	60% - 66%
C-	50% - 59%
I/F	less than 50%

## **Math Department Policies**

### **Respect, Participation, and Engagement**

Respectful conduct is expected throughout the class. The following are some indicators of the level of participation and engagement that is expected:

-behaving in a manner that reflects the school policies and expectations as outlined in the student handbook

-regular sharing of ideas and questions in class

-active and considerate listening

-active contributions to group work

-taking responsibility for catching up any missed work

-interacting with classmates and teachers in a respectful manner

### **Attendance**

Attendance and participation are necessary for success in this course. If you have to miss a class, please inform your teacher in advance. Please remember that learning is participatory, therefore, not attending classes will affect your learning of course content. Frequent absenteeism, obvious lack of familiarity with topics being dealt with in class, or lack of contribution to the work of the class will negatively impact your final grade.

### **Minimum Attendance Policy**

This is a semester long course, as such if you miss 8 blocks of instructional time during the course of the semester you will be accredited a grade of "W" for "Withdrawn" and will not be permitted to write any exams or receive a final grade in the course. Missed instructional time includes Unexcused Absences, Excused Absences, and Suspensions. Not counted are absences for documented illnesses and In School Suspensions.

If a student is assigned a "W" grade and wishes to reverse it, they must meet the following requirements:

· Complete all missing work in the course.

· Attend 1 hour of tutorial led by the teacher or in the Success Room after school in place of each block that was missed.

· Not miss any further instructional time in the course.

A student who has been assigned a "W" grade is still expected to attend and participate in all classes if they are pursuing a reversal of the "W" grade. After successfully completing their "W" grade recovery package, the absences recovered will be changed to a code of RA for Recovered Absent and will not count toward future absentee reports.

If the student does not wish to reverse the "W" grade or fails to follow the "W" grade recovery agreement set up for them, they will be withdrawn from the course and must attend a Skills and Planning block in place of that class for no credit.

### **Academic Dishonesty**

Academic is founded on honesty and integrity. As members of an academic institution, all students are expected to know, understand, and follow the codes of conduct regarding academic integrity. At the most basic level, this means submitting only original work done by you and acknowledging all sources of information or ideas and attributing them to others as required. This also means you should not cheat, copy, or mislead others about what is your work. Violations of academic integrity lead to serious consequences as outlined in the student handbook. For example, sharing information about tests/exams with other students, or recording information (photos) of tests/exams will result in parents being informed, written warning, and demerit recorded. Further violations will result in suspension and expulsion from the school.

### **Missed Assessments**

If you miss an assessment for any reason, you need to inform your teacher immediately. If you know that you will be missing an assessment before the day of that assessment, it is expected that you will let the teacher know so that they can make the appropriate arrangements with you. If you miss assessments too regularly, your teacher may arrange to notify your counsellor, your parents, the math department head and/or administrators in order to ensure that future assessments are not missed.

ARE YOU READY FOR CALCULUS?

1. Simplify: (a)  $\frac{x^3 - 9x}{x^2 - 7x + 12}$  (b)  $\frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$  (c)  $\frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$  (d)  $\frac{9 - x^{-2}}{3 + x^{-1}}$
2. Rationalize the denominator:  $\frac{2}{\sqrt{3} + \sqrt{2}}$  (b)  $\frac{4}{1 - \sqrt{5}}$  (c)  $\frac{1}{1 + \sqrt{3} - \sqrt{5}}$
3. Write each of the following expressions in the form  $ca^pb^q$  where  $c, p$  and  $q$  are numbers:

(a)  $\frac{(2a^2)^3}{b}$  (b)  $\sqrt{9ab^3}$  (c)  $\frac{a(2/b)}{3/a}$  (d)  $\frac{ab - a}{b^2 - b}$  (e)  $\frac{a^{-1}}{(b^{-1})\sqrt{a}}$  (f)  $\frac{a^2 \cdot 3}{b^{1/2}}$   $\frac{b^3 \cdot 2}{a^{1/2}}$

4. Solve for  $x$  (do not use a calculator):

(a)  $5^{(x+1)} = 25$  (b)  $\frac{1}{3} = 3^{2x+2}$  (c)  $\log_2 x = 3$  (d)  $\log_3 x^2 = 2 \log_3 4 - 4 \log_3 5$

5. Simplify: (a)  $\log_2 5 + \log_2(x^2 - 1) - \log_2(x - 1)$  (b)  $2 \log_4 9 - \log_2 3$  (c)  $3^{2 \log_3 5}$

6. Simplify: (a)  $\log_{10}(10^{1/2})$  (b)  $\log_{10}\left(\frac{1}{10^x}\right)$  (c)  $2 \log_{10} \sqrt{x} + 3 \log_{10} x^{1/3}$

7. Solve the following equations for the indicated variables:

(a)  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ , for  $a$  (b)  $V = 2(ab + bc + ca)$ , for  $a$

(c)  $A = 2\pi r^2 + 2\pi rh$ , for positive  $r$  (d)  $A = P + nrP$ , for  $P$

(e)  $2x - 2yd = y + xd$ , for  $d$  (f)  $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$ , for  $x$

8. For the equations (a)  $y = x^2 + 4x + 3$  (b)  $3x^2 + 3x + 2y = 0$  (c)  $9y^2 - 6y - 9 - x = 0$

complete the square and reduce to one of the standard forms  $y - b = A(x - a)^2$  or  $x - a = A(y - b)^2$ .

9. Factor completely: (a)  $x^6 - 16x^4$  (b)  $4x^3 - 8x^2 - 25x + 50$  (c)  $8x^3 + 27$  (d)  $x^4 - 1$

$x$ : (a)  $3 \sin^2 x = \cos^2 x$ ;  $0 \leq x < 2\pi$  (b)  $\cos^2 x - \sin^2 x = \sin x$ ;  $-\pi < x \leq \pi$

$x$   $x$   $x$ ;  $-\infty$   $\infty$

10. Find all real solutions to: (a)  $x^6 - 16x^4 = 0$  (b)  $4x^3 - 8x^2 - 25x + 50 = 0$  (c)  $8x^3 + 27 = 0$

11.

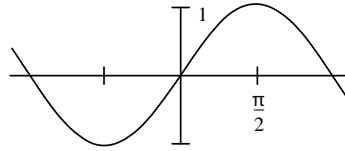
12. Solve for

(c)  $\tan x + \sec x = 2 \cos x$   $-\pi < x < \pi$

13. Without using a calculator, evaluate the following:

(a)  $\cos 210^\circ$  (b)  $\sin \frac{5}{4}$  (c)  $\tan^{-1}(-1)$  (d)  $\sin^{-1}(-1)$

(e)  $\cos \frac{9\pi}{4}$  (f)  $\sin^{-1} \frac{\sqrt{3}}{2}$  (g)  $\tan \frac{7\pi}{6}$  (h)  $\cos^{-1}(-1)$



14. Given the graph of  $\sin x$ , sketch the graphs of:

(a)  $\sin\left(x - \frac{\pi}{4}\right)$  (b)  $\sin\left(\frac{x}{2}\right)$  (c)  $2 \sin x$  (d)  $\cos x$  (e)  $\frac{1}{\sin x}$

15. Solve the equations: (a)  $x^2 + 12x + 3 = 0$  (b)  $2x + 1 = \frac{5}{x + 2}$  (c)  $\frac{x + 1}{x} - \frac{x}{x + 1} = 0$

16. Find the remainders on division of:

(a)  $x^5 - 4x^4 + x^3 - 7x + 1$  by  $x + 2$ . (b)  $x^5 - x^4 + x^3 + 2x^2 - x + 4$  by  $x^3 + 1$ .

17. (a) The equation  $12x^3 - 23x^2 - 3x + 2 = 0$  has a solution  $x = 2$ . Find all other solutions.

(b) Solve for  $x$ , the equation  $12x^3 + 8x^2 - x - 1 = 0$ . (All solutions are rational and between  $\pm 1$ .)

18. Solve the inequalities (a)  $x^2 + 2x - 3 \leq 0$  (b)  $\frac{2x - 1}{3x - 2} \leq 1$  (c)  $x^2 + x + 1 > 0$

19. Solve for  $x$ : (a)  $|-x + 4| \leq 1$  (b)  $|5x - 2| = 8$  (c)  $|2x + 1| = x + 3$

20. Determine the equations of the following lines: (a) the line through  $(-1, 3)$  and  $(2, -4)$ ;

(b) the line through  $(-1, 2)$  and perpendicular to the line  $2x - 3y + 5 = 0$ ; (c) the line through

$(2, 3)$  and the midpoint of the line segment from  $(-1, 4)$  to  $(3, 2)$ .

21. (a) Find the point of intersection of the lines:  $3x - y - 7 = 0$  and  $x + 5y + 3 = 0$

(b) Shade the region in the  $x - y$  plane that is described by the inequalities  $\begin{cases} 3x - y - 7 < 0 \\ x + 5y + 3 \geq 0 \end{cases}$ .

22. Find the equations of the following circles:

(a) the circle with centre at  $(1, 2)$  that passes through the point  $(-2, -1)$ ;

(b) the circle that passes through the origin and has intercepts equal to 1 and 2 on the  $x$ - and  $y$ - axes, respectively.

23. For the circle  $x^2 + y^2 + 6x - 4y + 3 = 0$ , find:

(a) the centre and radius; (b) the equation of the tangent at  $(-2, 5)$

24. A circle is tangent to the  $y$ -axis at  $y = 3$  and has one  $x$ -intercept at  $x = 1$ .

(a) Determine the other  $x$ -intercept. (b) Deduce the equation of the circle.

25. A curve is traced by a point  $P(x, y)$  which moves such that its distance from the point  $A(-1, 1)$  is three times its distance from the point  $B(2, -1)$ . Determine the equation of the curve.

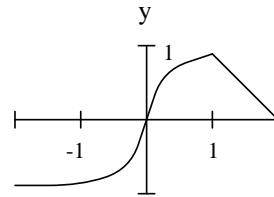
26. (a) Find the domain of the function.  $f(x) = \frac{3x + 1}{\sqrt{x^2 + x} - 2}$

(b) Find the domain and range of the functions: i)  $g(x) = \frac{5x - 3}{2x + 1}$

27. Let  $f(x) = \frac{|x|}{x}$ . Show that  $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$ . Find the domain and range of  $f(x)$ .

28. Simplify  $\frac{f(x+h) - f(x)}{h}$ , where (a)  $f(x) = 2x + 3$  (b)  $f(x) = \frac{1}{x+1}$  (c)  $f(x) = x^2$

29. The graph of the function  $y = f(x)$  is given as follows:



Determine the graphs of the functions:

- (a)  $f(x+1)$  (b)  $f(-x)$  (c)  $|f(x)|$  (d)  $f(|x|)$

30. Sketch the graphs of the functions: (a)  $g(x) = |3x + 2|$  (b)  $h(x) = |x(x - 1)|$

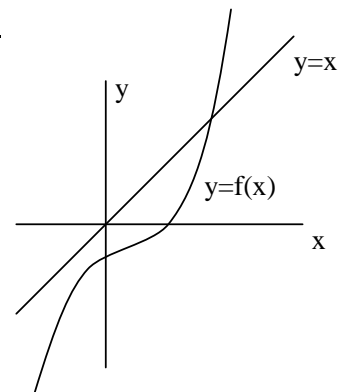
31. (a) The graph of a quadratic function (a parabola) has  $x$ -intercepts  $-1$  and  $3$  and a range consisting of all numbers less than or equal to  $4$ . Determine an expression for the function.

(b) Sketch the graph of the quadratic function  $y = 2x^2 - 4x + 3$ .

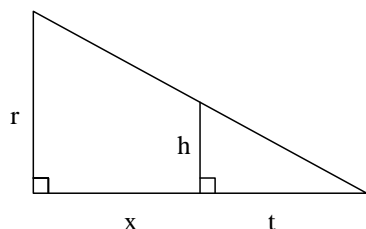
Write as a single equation in  $x$  and  $y$ : (a)  $\begin{cases} x = t+1 \\ y = t^2 - t \end{cases}$  (b)  $\begin{cases} x = \sqrt{t-1} \\ y = t^2 - t \end{cases}$  (c)  $\begin{cases} x = \sin t \\ y = \cos t \end{cases}$

32. Find the inverse of the functions: (a)  $f(x) = 2x + 3$  (b)  $f(x) = \frac{x+2}{5x-1}$  (c)  $f(x) = x^2 + 2x - 1, x > 0$

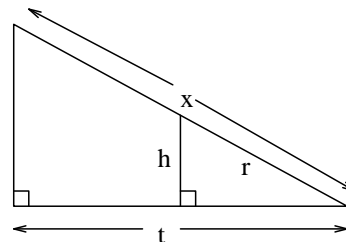
33. A function  $f(x)$  has the graph to the right. Sketch the graph of the inverse function  $f^{-1}(x)$ .



34. Express  $x$  in terms of the other variables in the picture.



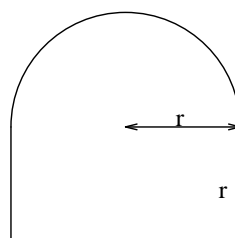
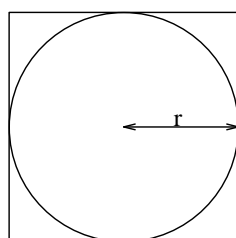
(a)



(b)

31.

35. (a) Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) below.



(a) (b)

(b) Find a formula for the perimeter of a window of the shape in the picture (b) above.

(c) A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface area of the top of the water?

- (d) Two cars start moving from the same point. One travels south at 100km/hour, the other west at 50 km/hour. How far apart are they two hours later?
- (e) A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal. (Assume that the string is perfectly straight.)

36. You should know the following trigonometric identities.

$$(A) \sin(-x) = -\sin x \quad (C) \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$(B) \cos(x) = \cos x \quad (D) \sin(x+y) = \sin x \cos y + \cos x \sin y$$

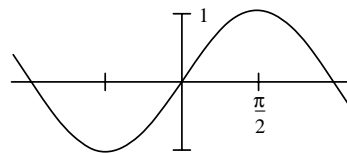
Use these to derive the following important identities, which you should also know.

$$(a) \sin^2 x + \cos^2 x \equiv 1 \text{ (use C and } \cos 0 = 1) \quad (b) \sin 2x \equiv 2 \sin x \cos x \quad (c) \cos 2x \equiv \cos^2 x - \sin^2 x$$

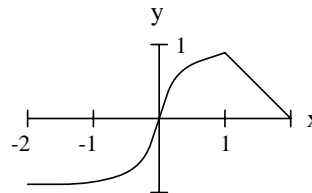
$$(d) \cos 2x \equiv 2 \cos^2 x - 1 \quad (e) \cos 2x \equiv 1 - 2 \sin^2 x \quad (f) \left| \cos \frac{x}{2} \right| \equiv \sqrt{\frac{1 + \cos x}{2}} \quad (g) \left| \sin \frac{x}{2} \right| \equiv \sqrt{\frac{1 - \cos x}{2}}$$

## ARE YOU READY FOR CALCULUS?

1. Simplify: (a)  $\frac{x^3 - 9x}{x^2 - 7x + 12}$  (b)  $\frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$  (c)  $\frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$  (d)  $\frac{9 - x^{-2}}{3 + x^{-1}}$
2. Rationalize the denominator: (a)  $\frac{2}{\sqrt{3} + \sqrt{2}}$  (b)  $\frac{4}{1 - \sqrt{5}}$  (c)  $\frac{1}{1 + \sqrt{3} - \sqrt{5}}$
3. Write each of the following expressions in the form  $ca^{pb^q}$  where  $c, p$  and  $q$  are numbers:  
 (a)  $\frac{(2a^2)^3}{b}$  (b)  $\sqrt{9ab^3}$  (c)  $\frac{a(2/b)}{3/a}$  (d)  $\frac{ab - a}{b^2 - b}$  (e)  $\frac{a^{-1}}{(b^{-1})\sqrt{a}}$  (f)  $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2 \left(\frac{b^{3/2}}{a^{1/2}}\right)$
4. Solve for  $x$  (do not use a calculator):  
 (a)  $5^{(x+1)} = 25$  (b)  $\frac{1}{3} = 3^{2x+2}$  (c)  $\log_2 x = 3$  (d)  $\log_3 x^2 = 2 \log_3 4 - 4 \log_3 5$
5. Simplify: (a)  $\log_2 5 + \log_2(x^2 - 1) - \log_2(x - 1)$  (b)  $2 \log_4 9 - \log_2 3$  (c)  $3^{2 \log_3 5}$
6. Simplify: (a)  $\log_{10}(10^{1/2})$  (b)  $\log_{10}\left(\frac{1}{10^x}\right)$  (c)  $2 \log_{10} \sqrt{x} + 3 \log_{10} x^{1/3}$
7. Solve the following equations for the indicated variables:  
 (a)  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ , for  $a$  (b)  $V = 2(ab + bc + ca)$ , for  $a$   
 (c)  $A = 2\pi r^2 + 2\pi rh$ , for positive  $r$  (d)  $A = P + nrP$ , for  $P$   
 (e)  $2x - 2yd = y + xd$ , for  $d$  (f)  $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$ , for  $x$
8. For the equations (a)  $y = x^2 + 4x + 3$  (b)  $3x^2 + 3x + 2y = 0$  (c)  $9y^2 - 6y - 9 - x = 0$   
 complete the square and reduce to one of the standard forms  $y - b = A(x - a)^2$  or  $x - a = A(y - b)^2$ .
9. Factor completely: (a)  $x^6 - 16x^4$  (b)  $4x^3 - 8x^2 - 25x + 50$  (c)  $8x^3 + 27$  (d)  $x^4 - 1$
10. Find all real solutions to: (a)  $x^6 - 16x^4 = 0$  (b)  $4x^3 - 8x^2 - 25x + 50 = 0$  (c)  $8x^3 + 27 = 0$
11. Solve for  $x$ : (a)  $3 \sin^2 x = \cos^2 x$ ;  $0 \leq x < 2\pi$  (b)  $\cos^2 x - \sin^2 x = \sin x$ ;  $-\pi < x \leq \pi$   
 (c)  $\tan x + \sec x = 2 \cos x$ ;  $-\infty < x < \infty$
12. Without using a calculator, evaluate the following:  
 (a)  $\cos 210^\circ$  (b)  $\sin \frac{5\pi}{4}$  (c)  $\tan^{-1}(-1)$  (d)  $\sin^{-1}(-1)$   
 (e)  $\cos \frac{9\pi}{4}$  (f)  $\sin^{-1} \frac{\sqrt{3}}{2}$  (g)  $\tan \frac{7\pi}{6}$  (h)  $\cos^{-1}(-1)$
13. Given the graph of  $\sin x$ , sketch the graphs of:  
 (a)  $\sin\left(x - \frac{\pi}{4}\right)$  (b)  $\sin\left(\frac{x}{2}\right)$  (c)  $2 \sin x$  (d)  $\cos x$  (e)  $\frac{1}{\sin x}$
14. Solve the equations: (a)  $4x^2 + 12x + 3 = 0$  (b)  $2x + 1 = \frac{5}{x+2}$  (c)  $\frac{x+1}{x} - \frac{x}{x+1} = 0$
15. Find the remainders on division of:  
 (a)  $x^5 - 4x^4 + x^3 - 7x + 1$  by  $x + 2$ . (b)  $x^5 - x^4 + x^3 + 2x^2 - x + 4$  by  $x^3 + 1$ .



16. (a) The equation  $12x^3 - 23x^2 - 3x + 2 = 0$  has a solution  $x = 2$ . Find all other solutions.  
 (b) Solve for  $x$ , the equation  $12x^3 + 8x^2 - x - 1 = 0$ . (All solutions are rational and between  $\pm 1$ .)
17. Solve the inequalities (a)  $x^2 + 2x - 3 \leq 0$  (b)  $\frac{2x-1}{3x-2} \leq 1$  (c)  $x^2 + x + 1 > 0$
18. Solve for  $x$ : (a)  $|-x + 4| \leq 1$  (b)  $|5x - 2| = 8$  (c)  $|2x + 1| = x + 3$
19. Determine the equations of the following lines: (a) the line through  $(-1,3)$  and  $(2,-4)$ ;  
 (b) the line through  $(-1,2)$  and perpendicular to the line  $2x - 3y + 5 = 0$ ;  
 (c) the line through  $(2,3)$  and the midpoint of the line segment from  $(-1,4)$  to  $(3,2)$ .
20. (a) Find the point of intersection of the lines:  $3x - y - 7 = 0$  and  $x + 5y + 3 = 0$   
 (b) Shade the region in the  $x - y$  plane that is described by the inequalities  $\begin{cases} 3x - y - 7 < 0 \\ x + 5y + 3 \geq 0 \end{cases}$ .
21. Find the equations of the following circles:  
 (a) the circle with centre at  $(1,2)$  that passes through the point  $(-2,-1)$ ;  
 (b) the circle that passes through the origin and has intercepts equal to 1 and 2 on the  $x$ - and  $y$ - axes, respectively.
22. For the circle  $x^2 + y^2 + 6x - 4y + 3 = 0$ , find:  
 (a) the centre and radius; (b) the equation of the tangent at  $(-2,5)$
23. A circle is tangent to the  $y$ -axis at  $y = 3$  and has one  $x$ -intercept at  $x = 1$ .  
 (a) Determine the other  $x$ -intercept. (b) Deduce the equation of the circle.
24. A curve is traced by a point  $P(x,y)$  which moves such that its distance from the point  $A(-1,1)$  is three times its distance from the point  $B(2,-1)$ . Determine the equation of the curve.
25. (a) Find the domain of the function  $f(x) = \frac{3x+1}{\sqrt{x^2+x-2}}$ .  
 (b) Find the domain and range of the functions: i)  $f(x) = 7$  ii)  $g(x) = \frac{5x-3}{2x+1}$
26. Let  $f(x) = \frac{|x|}{x}$ . Show that  $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$ . Find the domain and range of  $f(x)$ .
27. Simplify  $\frac{f(x+h) - f(x)}{h}$ , where (a)  $f(x) = 2x + 3$  (b)  $f(x) = \frac{1}{x+1}$  (c)  $f(x) = x^2$ .
28. The graph of the function  $y = f(x)$  is given as follows:  
 Determine the graphs of the functions:  
 (a)  $f(x+1)$  (b)  $f(-x)$  (c)  $|f(x)|$  (d)  $f(|x|)$

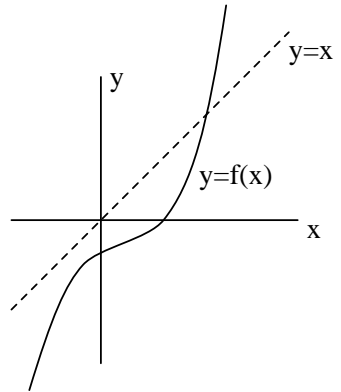




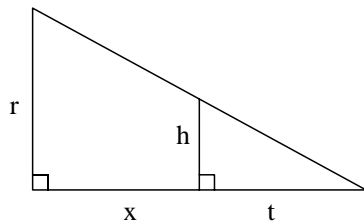
31. Write as a single equation in  $x$  and  $y$ : (a)  $\begin{cases} x = t + 1 \\ y = t^2 - t \end{cases}$  (b)  $\begin{cases} x = \sqrt[3]{t} - 1 \\ y = t^2 - t \end{cases}$  (c)  $\begin{cases} x = \sin t \\ y = \cos t \end{cases}$

32. Find the inverse of the functions: (a)  $f(x) = 2x + 3$  (b)  $f(x) = \frac{x + 2}{5x - 1}$   
 (c)  $f(x) = x^2 + 2x - 1, x > 0$

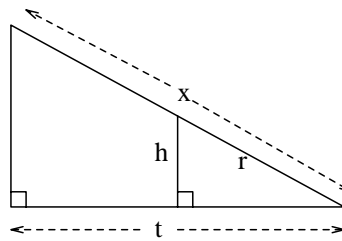
33. A function  $f(x)$  has the graph to the right  
 Sketch the graph of the inverse function  $f^{-1}(x)$ .



34. Express  $x$  in terms of the other variables in the picture.

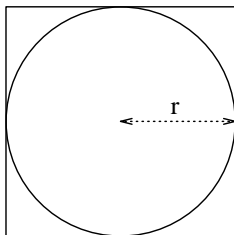


(a)

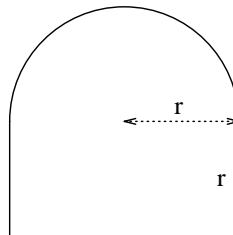


(b)

35. (a) Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) below.



(a)



(b)

- (b) Find a formula for the perimeter of a window of the shape in the picture (b) above.  
 (c) A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface area of the top of the water?  
 (d) Two cars start moving from the same point. One travels south at 100km/hour, the other west at 50 km/hour. How far apart are they two hours later?  
 (e) A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal. (Assume that the string is perfectly straight.)

36. You should know the following trigonometric identities.

(A)  $\sin(-x) = -\sin x$  (C)  $\cos(x + y) = \cos x \cos y - \sin x \sin y$   
 (B)  $\cos(-x) = \cos x$  (D)  $\sin(x + y) = \sin x \cos y + \cos x \sin y$

Use these to derive the following important identities, which you should also know.

(a)  $\sin^2 x + \cos^2 x \equiv 1$  (use C and  $\cos 0 = 1$ ) (b)  $\sin 2x \equiv 2 \sin x \cos x$  (c)  $\cos 2x \equiv \cos^2 x - \sin^2 x$   
 (d)  $\cos 2x \equiv 2 \cos^2 x - 1$  (e)  $\cos 2x \equiv 1 - 2 \sin^2 x$  (f)  $\left| \cos \frac{x}{2} \right| \equiv \sqrt{\frac{1 + \cos x}{2}}$  (g)  $\left| \sin \frac{x}{2} \right| \equiv \sqrt{\frac{1 - \cos x}{2}}$