CHAPTER

3

A projectile is an object moving in a vertical plane, under the influence of the force of gravity.

Learning Expectations

By the end of this chapter, you will:

Relating Science to Technology, Society, and the Environment

- analyze, on the basis of research, a technology that applies concepts related to kinematics

Developing Skills of Investigation and Communication

- conduct an inquiry into the uniform and non-uniform linear motion of an object
- solve problems involving uniform and non-uniform linear motion in one and two dimensions, using graphical analysis and algebraic equations
- use kinematic equations to solve problems related to the horizontal and vertical components of the motion of a projectile
- conduct an inquiry into the projectile motion of an object, and analyze, in qualitative and quantitative terms, the relationship between the horizontal and vertical components

Understanding Basic Concepts

- describe the characteristics and give examples of a projectile's motion in vertical and horizontal planes

On Canada Day, spectacular fireworks shows can be seen throughout the country (Figure 3.1). A firework is the combination of a firecracker and a sparkler. The firecracker produces the explosion and the sparkler produces the extremely bright light with colourful sparks of light. The different colours are the result of the different chemical compounds used in the firework. The path of each spark is two dimensional. It is a combination of both vertical and horizontal directions.

Figure 3.1 Many people enjoy watching fireworks displays on holidays such as Canada Day.

3.1 Acceleration due to Gravity — Free Fall

Section Summary

- An object falling only under the influence of gravity undergoes "free fall."
- An object falls with a constant vertical acceleration equal to *g*.
- The acceleration and velocity of a free falling object do not depend on mass of the object.
- The velocity of an object thrown vertically upwards will be equal and opposite to its velocity when it falls back to its initial point.

Many amusement parks and midways showcase a ride based solely on acceleration due to gravity. The ride transports thrill seekers up to a dizzying height, allows them to come to rest, and then, without warning, releases them downward before coming to a controlled stop (Figure 3.2).

In the previous chapters, you learned about objects that move in a horizontal plane and objects that move in a vertical plane. Flipping a coin, kicking a football, and a free throw in basketball are all examples of objects experiencing motion in a vertical plane (Figure 3.3). This type of motion is called **projectile motion**. A **projectile** is any object thrown into the air. Projectiles include objects dropped from rest; objects thrown downward or vertically upward, such as a tennis serve toss; and objects projected horizontally or moving upward at an angle, such as a punted football.

In chapter 2 you saw that mass has no effect on the motion or acceleration of a falling object (Figure 3.4). This constant acceleration is called **acceleration due to gravity**, or *g*. The value of *g* near Earth's surface is 9.81 m/s^2. This value is only approximate because the acceleration due to gravity varies slightly at different locations on Earth as a result of differences in elevation. Air resistance is another factor that reduces acceleration due to gravity. Both these effects will be ignored here for simplicity. In the 16th century, Galileo Galilei (1564–1642) conducted experiments that clearly demonstrated that objects falling near Earth's surface fall with a constant acceleration, once the effects of air resistance are removed.

Figure 3.2 Amusement park rides are an application of physics

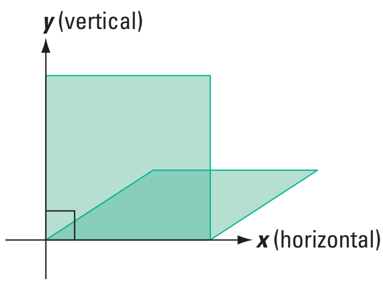

Figure 3.3 A plane has two dimensions, *x* and *y*.

Figure 3.4 A demonstration with multi-flash photography: an apple and a feather are released simultaneously through a trap door in a large vacuum chamber and they fall together — almost. Because the chamber has partial vacuum, there is still some air resistance

Gravity Causes Objects to Accelerate Downward

Objects in motion only under the influence of gravity are said to be in **free fall**. The kinematics equations in one dimension that were derived in the last chapter can be modified for free fall. The acceleration for all the equations will be equal to $\vec{g}$, which is a vector quantity. The direction of $\vec{g}$ is always vertically downwards and its magnitude is 9.81 m/s².

An Object Falling Down

If you drop a golf ball from a height of 1.25 m, how long will it take for the ball to reach the ground (Figure 3.5)? Because the ball is moving in only one direction, down, choose down to be positive for simplicity. Since the golf ball is accelerating due to gravity starting from rest,

$$\vec{v}_i = 0 \text{ and } \vec{a} = 9.81 \text{ m/s}^2 \text{ [down]} = +9.81 \text{ m/s}^2$$

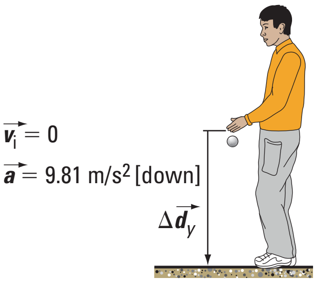

Figure 3.5 The time it takes the golf ball to hit the ground depends on the height from which it is dropped and the acceleration due to gravity.

The ball's displacement can be expressed as 1.25 m [down], or + 1.25 m. The equation that includes all the given variables and the unknown variable is $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$. The displacement and acceleration vectors are both in the same direction, so use the scalar form of the equation to solve for time. Since $\vec{v}_i = 0$,

$$\Delta d = (0)\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$\Delta d = \frac{1}{2}a(\Delta t)^2$$

$$(\Delta t)^2 = \frac{2\Delta d}{a}$$

$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

$$= \sqrt{\frac{2(1.25\ \cancel{\text{m}})}{9.81\ \cancel{\text{m}}/\text{s}^2}}$$

$$= 0.505 \text{ s}$$

The golf ball takes 0.505 s to reach the ground when released from a rest height of 1.25 m.

Note that the time it takes for an object to fall is directly proportional to the square root of the height it is dropped from: $\Delta t = \sqrt{\frac{2\Delta\vec{d}}{\vec{a}}}$. If there is no air resistance, the time it takes for a falling object to reach the ground depends only on the height from which it was dropped. The time does not depend on any other property (such as mass, volume, or density) of the object. However, measuring time using certain equipment could involve uncertainties. This is because people take time to see an action and think about it before they are able to react to it. This time is called the reaction time. This time increases if the situation is more complex. Moreover, every person has a different reaction time which can be reduced by practicing or repeating the same situation. For example, when the light turns green at a traffic signal, every driver takes a few seconds to react and start driving again.

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Explore More

A ball is thrown vertically downwards from the top of a cliff. Is the ball in free fall?

Instead of dropping an object ($\vec{v}_i = 0$) such as a golf ball, what if you threw an object down ($\vec{v}_i \neq 0$)? By throwing an object straight down, you give the object an initial vertical velocity downward. What effect does an initial velocity have on the motion of the object? The next example will show you.

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Suggested Activity

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Example 3.1

While cliff diving in Mexico, a diver throws a smooth, round rock straight down with an initial speed of 4.00 m/s. If the rock takes 2.50 s to land in the water, how high is the cliff?

Given

For convenience, choose down to be positive because down is the only direction of the rock's motion.

$v_i = 4.00$ m/s [down] $= +4.00$ m/s

$\Delta t = 2.50$ s

$\vec{a} = 9.81$ m/s² [down] $= +9.81$ m/s²

Required

height of cliff (Δd)

Analysis and Solution

The initial velocity and acceleration vectors are both in the same direction, so use the scalar form of the equation $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$.

$$\begin{aligned}\Delta d &= (4.00\text{ m/s})(2.50\text{ s}) + \frac{1}{2}(9.81\text{ m/s}^2)(2.50\text{ s})^2 \\ &= 10.0\text{ m} + 30.7\text{ m} \\ &= 40.7\text{ m}\end{aligned}$$

Paraphrase

The cliff is 40.7 m high.

Practice Problems

1. If a rock takes 0.750 s to hit the ground after being thrown down from a height of 4.80 m, determine the rock's initial velocity.
2. Having scored a touchdown, a football player spikes the ball in the end zone. If the ball was thrown down with an initial velocity of 2.0 m/s from a height of 1.75 m, determine how long until it hits the ground.
3. An elevator moving downward at 4.00 m/s experiences an upward acceleration of 2.00 m/s² for 1.80 s. What is its velocity at the end of the acceleration and how far has it travelled?

Answers

1. 2.72 m/s [down]
2. 0.43 s
3. 0.400 m/s [down], 3.96 m

What Goes Up Must Come Down

Circus clowns are often accomplished jugglers (Figure 3.6). If a juggler throws a ball upward, giving it an initial velocity, what happens to the ball (Figure 3.7)?

When you throw an object up, its height (displacement) increases while its velocity decreases. The decrease in velocity occurs because the object experiences acceleration downward due to gravity (Figure 3.8(a) on the next page). The ball reaches its maximum height when its vertical velocity equals zero. In other words, it stops for an instant at the top of its path (Figure 3.8(b)). But remember, even if the velocity is zero at this point, the acceleration here is still the same (g acting vertically downwards). This acceleration is the reason that the object starts falling downwards. When the object falls back toward the ground, it speeds up because its velocity and acceleration are now in the same direction (Figure 3.8(c)).

Figure 3.6 Juggling is an example of projectile motion.

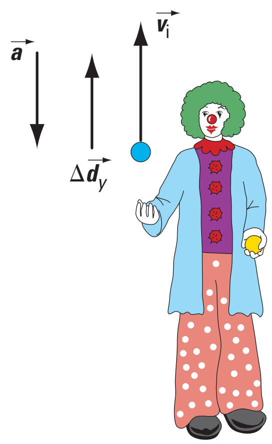

Figure 3.7 The ball's motion is called vertical projectile motion.

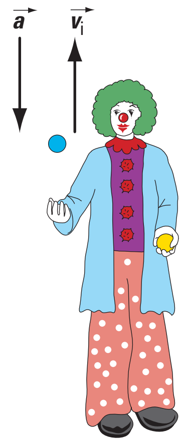

Figure 3.8(a) Stage 1: Velocity and acceleration are in opposite directions, so the ball slows down.

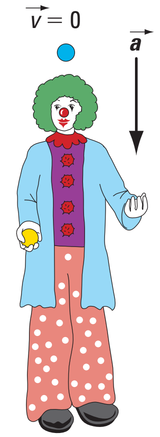

Figure 3.8(b) Stage 2: The ball has momentarily stopped, but its acceleration is still 9.81 m/s² [down], which causes the ball to change direction.

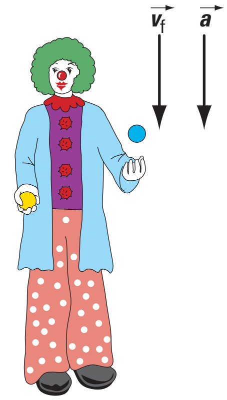

Figure 3.8(c) Stage 3: Velocity and acceleration are in the same direction, so the ball speeds up.

The next two examples analyze different stages of the same object's motion. Example 3.2 analyzes the upward part of the motion of an object thrown upward, whereas Example 3.3 analyzes the same object's downward motion.

Example 3.2

A clown throws a ball upward at 10.00 m/s. Find
(a) the maximum height the ball reaches above its launch height
(b) the time it takes to do so

Given
Consider up to be positive.
$\vec{v}_i = 10.00$ m/s [up] $= +10.00$ m/s $\vec{a} = 9.81$ m/s² [down] $= -9.81$ m/s²

Required
(a) maximum height above launch height (Δd)
(b) time taken to reach maximum height (Δt)

Analysis and Solution
(a) When you throw an object up, as its height increases, its speed decreases because the object is accelerating downward due to gravity. The ball, travelling upward away from its initial launch height, reaches its maximum height when its vertical velocity is zero. In other words, the object stops for an instant at the top of its path, so $v_f = 0.00$ m/s. To find the object's maximum height, neglecting air friction, use the equation $v_f^2 = v_i^2 + 2a\Delta d$ and substitute scalar quantities.

$$\begin{aligned}\Delta d &= \frac{v_f^2 - v_i^2}{2a}\\ &= \frac{(0.00\ \text{m/s})^2 - (10.00\ \text{m/s})^2}{2(-9.81\ \text{m/s}^2)}\\ &= 5.10\ \text{m}\end{aligned}$$

Practice Problems

1. A girl tosses a volleyball upward at 6.0 m/s.
   (a) What is the maximum height the ball reaches above its launch height?
   (b) How long does it take to reach its maximum height?
2. The Drop Zone drops riders 27 m from rest before slowing them down to a stop. How fast are they moving before they start slowing down?

Answers

1. (a) 1.8 m
   (b) 0.61 s
2. 23 m/s

(b) To find the time taken, use the equation $v_f = v_i + a\Delta t$. Substitute scalar quantities because you are dividing vectors.

$$v_f = v_i + a\Delta t$$
$$\Delta t = \frac{v_f - v_i}{a}$$
$$= \frac{0.00 \text{ m/s} - 10.00 \text{ m/s}}{-9.81 \text{ m/s}^2}$$
$$= 1.02 \text{ s}$$

Paraphrase

(a) The ball's maximum height is 5.10 m above its launch height.

(b) It takes the ball 1.02 s to reach maximum height.

The next example is a continuation of the previous example. It analyzes the same ball's motion as it falls back down from its maximum height.

Example 3.3

A clown throws a ball upward at 10.00 m/s. Find

(a) the time it takes the ball to return to the clown's hand from maximum height

(b) the ball's final velocity

Given

Consider up to be positive.

$\vec{v}_i = 10.00 \text{ m/s [up]} = +10.00 \text{ m/s}$

$\vec{a} = 9.81 \text{ m/s}^2 \text{ [down]} = -9.81 \text{ m/s}^2$

Required

(a) time taken to land (Δt)

(b) final velocity ($\vec{v}_f$)

Analysis and Solution

(a) For an object starting from rest at maximum height and accelerating downward due to gravity, its motion is described by the equation $\Delta\vec{d} = v_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$ where $\vec{v}_i = 0$ (at maximum height). For downward motion, the ball's displacement and acceleration are in the same direction, so use the scalar form of the equation. For Δd, substitute 5.10 m (from the previous example). Rearrange this equation and substitute the values.

$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$
$$= \sqrt{\frac{2(5.10 \cancel{\text{m}})}{9.81 \cancel{\text{m}}/\text{s}^2}}$$
$$= 1.02 \text{ s}$$

(b) The ball's final velocity (starting from maximum height) when it lands on the ground is

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$
$$= 0.00 \text{ m/s} + (-9.81 \text{ m/s}^2)(1.02 \text{ s})$$
$$= -10.0 \text{ m/s}$$

The negative sign means that the direction is downward.

Paraphrase

(a) It takes the ball 1.02 s to return to the clown's hand.

(b) The final velocity at the height of landing is 10.0 m/s [down].

Practice Problems

1. A pebble falls from a ledge 20.0 m high.
   (a) Find its velocity just before it hits the ground.
   (b) Find the time it takes to hit the ground.

Answers

1. (a) 19.8 m/s [down]
   (b) 2.02 s

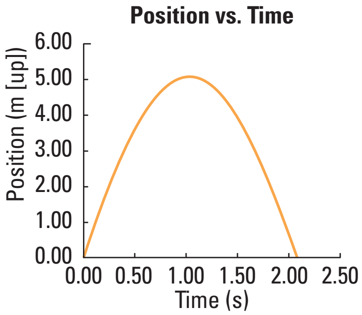

Figure 3.9 The position-time graph of a ball thrown vertically upward is a parabola.

You can use the data calculated in Examples 3.2 and 3.3 to plot a position-time graph of the ball's motion. Because the ball experiences uniformly accelerated motion, the graph is a parabola (Figure 3.9).

A Graphical Representation of a Vertical Projectile

You can now represent the motion of the juggler's ball on a position-time graph. Remember that the ball's motion can be divided into three different stages: Its speed decreases, becomes zero, and then increases. However, the velocity is uniformly decreasing. The graphs that correspond to these three stages of motion are shown in Figure 3.10.

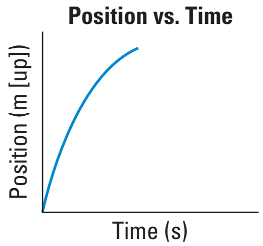

Figure 3.10(a) Consider up to be positive. The ball rises until it stops.

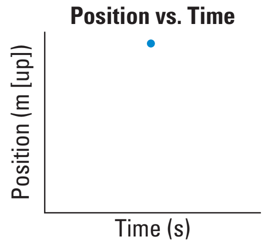

Figure 3.10(b) The ball stops momentarily at maximum height.

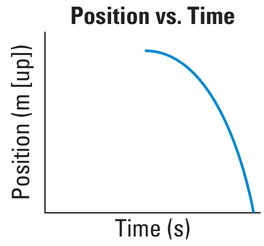

Figure 3.10(c) The ball falls back down to its launch height.

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Suggested Activity

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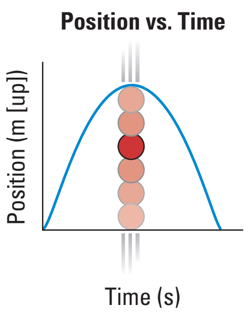

Figure 3.11 A ball thrown straight up in the air illustrates uniformly accelerated motion.

Now put these three graphs together to generate the complete position-time graph of the ball's motion (Figure 3.11). Remember that the ball is actually moving straight up and down, and not in a parabolic path.

To generate a corresponding velocity-time graph from the position-time graph in Figure 3.11, draw a series of tangents at specific time instances. Choosing strategic points will make your task easier. The best points to choose are those that begin and end a stage of motion because they define that stage (Figure 3.12(a)).

In Figure 3.12(a), notice that the initial slope of the tangent on the position-time graph is positive, corresponding to an initial positive (upward) velocity on the velocity-time graph below (Figure 3.12(b)). The last tangent has a negative slope, corresponding to a final negative velocity on the velocity-time graph. The middle tangent is a horizontal line (slope equals zero), which means that the ball stopped momentarily. Remember that the slope of a velocity-time graph represents acceleration.

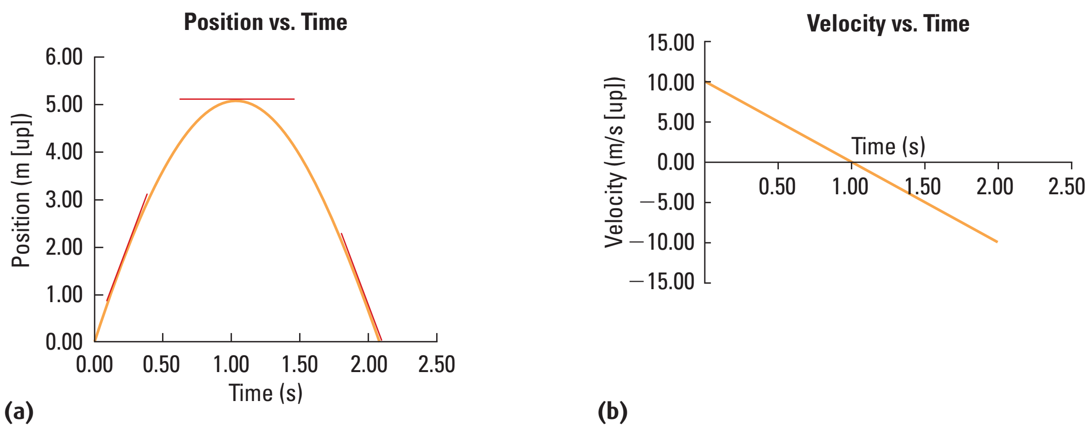

Figure 3.12 To generate the velocity-time graph in (b) corresponding to the position-time graph in (a), draw tangents at strategic points.

Concept Check

1. What should be the value of the slope of the velocity-time graph for vertical projectile motion?
2. For the velocity-time graph of an experiment of a vertical throw, will the slope be equal to the expected value? Explain.
3. Is acceleration due to gravity the same at all locations on Earth?

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Take It Further

Can you shoot an object fast enough so that it does not return to Earth? Research escape velocity. Is it the same regardless of the size of an object? How do you calculate it? Write a brief summary of your findings.

A9 Quick Lab

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Could You Be a Goalie for the NHL?

Purpose

To measure your reaction time

Activity Overview

Reaction times are critical for some occupations, such as an NHL goalie. In this activity, you and your partner will calculate each of your reaction times by grasping a dropped ruler. Figure 3.13 shows you the setup for this activity.

Your teacher will give you a copy of the full activity.

Prelab Questions

Consider the questions below before beginning this activity.

1. Give three examples in sports, other than hockey, where reaction time is important.
2. Give three examples in day to day activities where reaction time is important.
3. What conditions can cause a decline in reaction time?

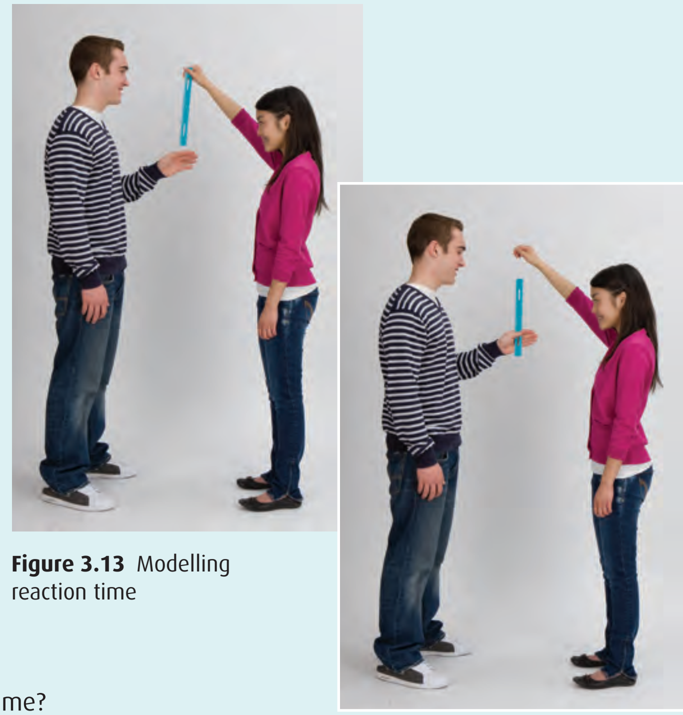

Figure 3.13 Modelling reaction time

A10 Quick Lab

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Using Accelerometers

Purpose

To research and report on how an accelerometer can be used to help patients with balance disorders

Activity Overview

In this activity, you will research accelerometers. You will write a report explaining how the concepts of kinematics are applied to the technology.

Your teacher will give you a copy of the full activity.

Prelab Questions

Consider the questions below before beginning this activity.

1. How does an accelerometer work?
2. What are the most common uses of an accelerometer?

3.1 Check and Reflect

Key Concept Review

1. Define a projectile.

2. What determines how long it will take an object to reach the ground when released with an initial velocity of zero?

3. The position-time graph of a ball thrown vertically up is a parabola. Describe the velocity of the ball
(a) on its way up
(b) when it reaches its maximum height
(c) on its way down

4. A child throws a soft toy ball vertically upward at 8.0 m/s. It takes 3.3 s to reach its maximum height.
(a) How long does it take the ball to return to the child's hand? What assumption did you make?
(b) What is the ball's final velocity?

Connect Your Understanding

5. A student drops a bran muffin from the roof of the school. From what height is the muffin dropped if it hits the ground 3.838 s later?

6. During a babysitting assignment, a babysitter is constantly picking up toys dropped from the infant's highchair. If the toys drop from rest and hit the floor 0.56 s later, from what height are they being dropped?

7. A rock takes 1.575 s to drop 2.00 m down toward the surface of the Moon. Determine the acceleration due to gravity on the Moon.

8. At the beginning of a game, a referee throws a basketball vertically upward with an initial speed of 5.0 m/s. Determine the maximum height above the floor reached by the basketball if it starts from a height of 1.50 m.

9. A student rides West Edmonton Mall's Drop of Doom. If the student starts from rest and falls due to gravity for 2.6 s, what will be his final velocity and how far will he have fallen?

10. If the acceleration due to gravity on Jupiter is 24.8 m/s^2 [down], determine the time it takes for a tennis ball to fall 1.75 m from rest.

11. If a baseball popped straight up into the air has a hang time (length of time in the air) of 6.25 s, determine the distance from the point of contact to the baseball's maximum height. Assume the ball is caught at the same height it is hit.

12. A kangaroo jumps from the ground level vertically upwards to a height of 3.0 m. How long does it take the kangaroo to come back to the ground?

13. A penny is dropped from a cliff of height 190 m. Determine the time it takes for the penny to hit the bottom of the cliff.

14. A coin tossed straight up into the air takes 2.75 s to go up and down to its initial release point 1.30 m above the ground. What is its maximum height above the ground?

15. If a diver starts from rest, determine the amount of time he takes to reach the water's surface from the 10-m platform.

16. John drops a key from a window which is 5.0 m above a point on the ground below. At the same time, Jane starts walking from a distance towards this point on the ground with a speed of 2.75 m/s. When she reaches this point, she catches the key when it is 1.25 m above the ground. How far was Jane from the point when the key was dropped?

17. A rocket launched vertically upward accelerates uniformly for 50 s until it reaches a velocity of 200 m/s [up]. At that instant, its fuel runs out.
(a) Calculate the rocket's initial acceleration.
(b) Calculate the height of the rocket when its fuel runs out.
(c) Explain why the rocket continues to gain height for 20 s after its fuel runs out.
(d) Calculate the maximum height of the rocket.

18. A ball is dropped from a height of 60.0 m. A second ball is thrown down from the same height 0.850 s later. If both balls reach the ground at the same time, what was the initial velocity of the second ball?

Reflection

19. After learning about free fall, did your opinion about how long it takes an object to travel up, compared to how long it takes it to fall back down change? How?

For more questions, go to PHYSICS•SOURCE

3.2 Two Dimensional Vectors in Motion

Section Summary

- Projectiles are objects undergoing two dimensional motion under the influence of gravity
- Two dimensional vectors can be resolved into *x* and *y* components
- Two or more vectors can be added by using components

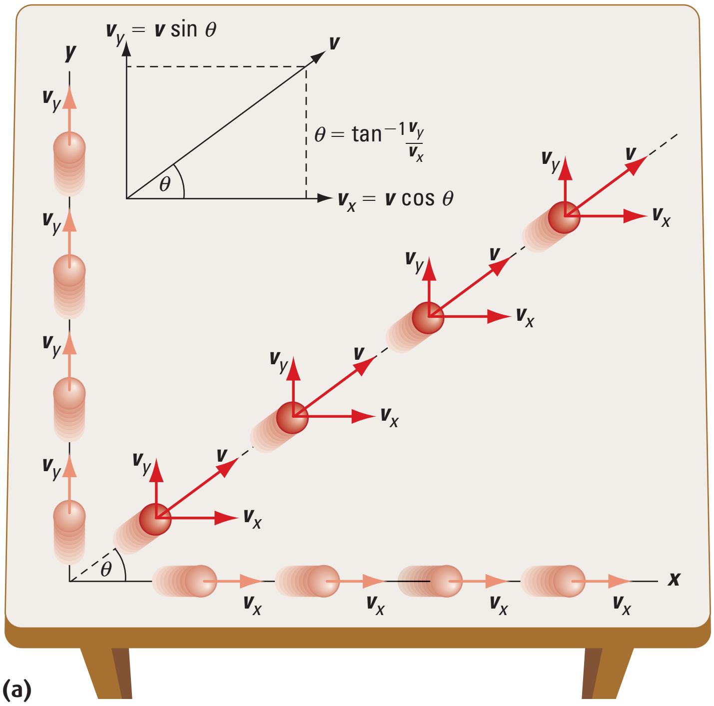
Whether it is golf, ball hockey, volleyball, basketball, or soccer, many sports involve a ball being shot in the air (Figure 3.14). The ball is a projectile flying through the air under the influence of gravity. The sparks from fireworks or welding are other examples of projectiles. These projectiles are displaced simultaneously in the horizontal and the vertical directions. Such motion is called two dimensional motion, and it can be analyzed using vector mathematics that you learned in the first chapter. The kinematic equations written for both the horizontal and vertical directions are used for this analysis. We will assume that air resistance is negligible.

Two dimensional motion is observed all the time in our lives. A ball moving on a pool table or you running across a field diagonally, are examples of such motion.

Figure 3.14 Motion in sports can be described by vectors in two dimensions.

Components of Vectors

If you are playing pool, consider the motion of the ball moving diagonally across the table as shown in Figure 3.15(a). The ball reaches the other side of the diagonal. To help understand this motion, imagine an x-axis and a y-axis along the two sides of the table as shown in Figure 3.15(b). The diagonal motion vector can then be separated, or resolved, into two perpendicular parts, or **components**: the x component and the y component. The diagonal vector is the resultant vector. The ball can reach the other side of the table by first moving along the x-axis and then along the y-axis. We can say that the two paths are equivalent to each other because the displacement of the ball is the same in both cases.



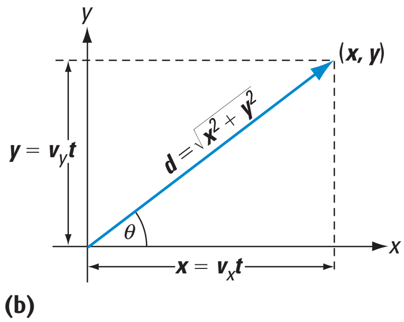


Figure 3.15 (a) The velocity for uniform, straight line motion may have x and y components because of the chosen orientation of the coordinate axes. (b) The ball's position and the distance d the ball has travelled from the origin can be found at any time t.

The angle that the diagonal makes with the x-axis is the Greek letter theta, θ. In Figure 3.15(b), the length of the diagonal d is related to the two sides (d_x and d_y) as

$$\cos\theta = \frac{d_x}{d} \text{ and, } \sin\theta = \frac{d_y}{d}$$

Then

$$d_x = d\cos\theta$$
$$d_y = d\sin\theta$$

Now, the velocity along these two directions will be

$$v_x = \frac{d_x}{t} \text{ and } v_y = \frac{d_y}{t}$$

$$d_x = v_x t \text{ and } d_y = v_y t$$

From the above two equations for d_x and d_y, we get,

$$d\cos\theta = v_x t \text{ and } d\sin\theta = v_y t$$

$$\frac{d}{t}\cos\theta = v_x \text{ and } \frac{d}{t}\sin\theta = v_y$$

which gives us the two equations for the x and y components of velocity

$v_x = v \cos \theta$

$v_y = v \sin \theta$

Using Pythagoras theorem, we also get

$$d = \sqrt{d_x^2 + d_y^2} \quad \text{and} \quad \tan \theta = \frac{d_y}{d_x}$$

$$v = \sqrt{v_x^2 + v_y^2} \qquad \tan \theta = \frac{v_y}{v_x}$$

In the above method, you learned how to find the x and y components for a vector. In some cases (as shown in the next example), the x-axis may represent the East/West directions and the y-axis, the North/South directions.

Example 3.4

Determine the north and east velocity components of a car travelling at 100 km/h [25° N of E] (Figure 3.16).

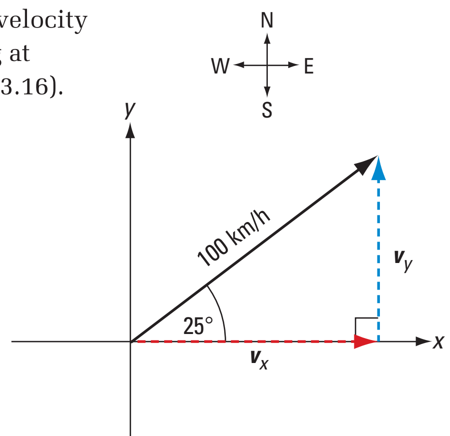


Figure 3.16

Given

$\vec{v}$ = 100 km/h [25° N of E]

Required

north component of velocity (y component, v_y)

east component of velocity (x component, v_x)

Analysis and Solution

The vector lies between the north and east directions, the angle it makes with the x-axis is 25°.

The north component:

$$\begin{aligned} v_y &= v \sin \theta \\ &= (100 \text{ km/h})(\sin 25°) \\ &= 42.3 \text{ km/h} \end{aligned}$$

The east component:

$$\begin{aligned} v_x &= v \cos \theta \\ &= (100 \text{ km/h})(\cos 25°) \\ &= 90.6 \text{ km/h} \end{aligned}$$

Paraphrase

The north component of the car's velocity is 42.3 km/h and the east component is 90.6 km/h.

Practice Problems

1. A hiker's displacement is 15 km [40° E of N]. What is the north component of his displacement?
2. A cyclist's velocity is 10 m/s [245°]. Determine the x and y components of her velocity.
3. A snowmobile travels 65 km [37° E of S]. How far east does it travel?

Answers

1. 11 km [N]
2. $v_x = -4.2$ m/s, $v_y = -9.1$ m/s
3. 39 km [E]

Adding Vectors Using Components

You can write the magnitude of any two-dimensional vector as the sum of its x and y components. Note that x and y components are perpendicular. Because motion along the x direction is perpendicular to motion along the y direction, a change in one component does not affect the other component. Whether it is movement across a pool table or any type of two-dimensional motion, you can describe the motion in terms of x and y components.

In general, most vector motion involves adding non-collinear vectors. Non-collinear vectors are vectors that are not along the same straight line. Consider the following scenario. During a lacrosse game, players pass the ball from one person to another (Figure 3.17). The ball can then be redirected for a shot on goal. Each of the displacements could involve different angles. In order to find the net displacement, you would use the following sequence of calculations.

Figure 3.17 The movement of the players and the ball in a lacrosse game could be tracked using vectors.

PHYSICS•SOURCE

Suggested Activity

- A12 Quick Lab Overview on page 74

Four Steps for Adding Non-collinear Vectors Algebraically

1. Determine the x and y components of each vector.
2. Add all the components in the x direction. Add all the components in the y direction. The sums of the x and y components are the two (perpendicular) components of the resultant vector.
3. To find the magnitude of the resultant vector, use the Pythagorean theorem.
4. Find the angle of the resultant vector using trigonometric ratios.

The following example illustrates how to apply these steps.

In a lacrosse game (Figure 3.18(a)), player A passes the ball 12.0 m to player B at an angle of 30°. Player B relays the ball to player C, 9.0 m away, at an angle of 155°. Find the ball's resultant displacement.

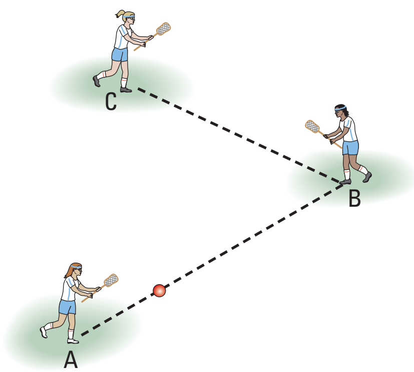


Figure 3.18(a) The path of the ball on the lacrosse field

Figure 3.18(b) shows the path of the lacrosse ball as vectors. This problem is different from previous examples because the two vectors are not at right angles to each other. Even with this difference, you can follow the same general steps to solve the problem.

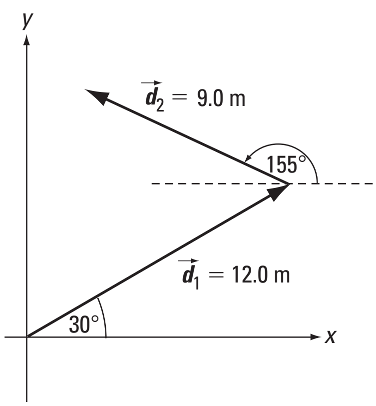


Figure 3.18(b) The path of the ball as vectors

Step 1: Determine the x and y components of each vector.
Since you are solving for displacement, resolve each displacement vector into its components (Figure 3.19). Table 3.1 shows how to calculate the x and y components. In this case, designate up and right as positive directions.

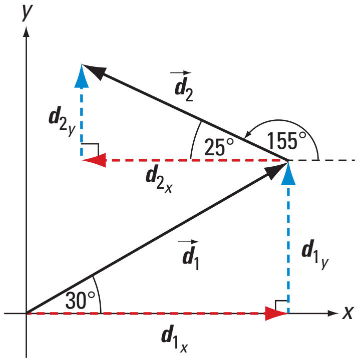


Figure 3.19 The components of each displacement vector

Table 3.1 Resolution of components in Figure 3.19

***x* direction**
$d_{1_x} = (12.0\text{ m})(\cos 30°)$ $= 10.39\text{ m}$ $d_{2_x} = (9.0\text{ m})(\cos 25°)$ $= -8.16\text{ m}$
***y* direction**
$d_{1_y} = (12.0\text{ m})(\sin 30°)$ $= 6.00\text{ m}$ $d_{2_y} = (9.0\text{ m})(\sin 25°)$ $= 3.80\text{ m}$

(Note that d_{2_x} is negative because it points to the left. Up and right were designated positive.)

Step 2: Add the x components and the y components separately.
Add all the x components together, then add all the y components together (Figure 3.20 and Table 3.2).

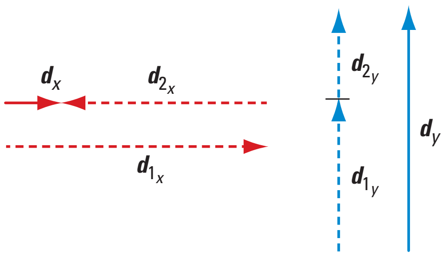


Figure 3.20 Add the *x* and *y* components separately first to obtain two perpendicular vectors.

Table 3.2 Adding *x* and *y* components in Figure 3.20

***x* direction**
$d_x = d_{1_x} + d_{2_x}$ $= 10.39\text{ m} + (-8.16\text{ m})$ $= 10.39\text{ m} - 8.16\text{ m}$ $= 2.23\text{ m}$
***y* direction**
$d_y = d_{1_y} + d_{2_y}$ $= 6.00\text{ m} + 3.80\text{ m}$ $= 9.80\text{ m}$

Step 3: Find the magnitude of the resultant, $\vec{d}$.
To find the magnitude of the resultant, use the Pythagorean Theorem (Figure 3.21).

$$d^2 = (d_x)^2 + (d_y)^2$$
$$d = \sqrt{(d_x)^2 + (d_y)^2}$$
$$= \sqrt{(2.23\text{ m})^2 + (9.80\text{ m})^2}$$
$$= 10.1\text{ m}$$

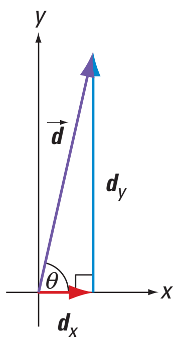


Figure 3.21 The component method allows you to convert non-perpendicular vectors into perpendicular vectors that you can then combine using the Pythagorean theorem.

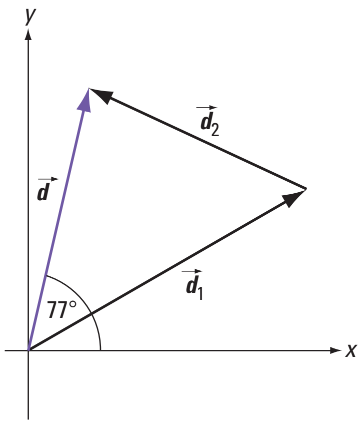


Figure 3.22 $\vec{d}$ is the resultant displacement of the ball.

Step 4: Find the angle of $\vec{d}$.
Use the tangent function to find the angle of the resultant (Figure 3.21).

$$\begin{aligned}\tan\theta &= \frac{d_y}{d_x}\\ &= \frac{9.80\ \cancel{\text{m}}}{2.23\ \cancel{\text{m}}}\\ &= 4.39\\ \theta &= \tan^{-1}(4.39)\\ &= 77°\end{aligned}$$

The ball's displacement is, therefore, 10.1 m [77°], as shown in Figure 3.22.

Example 3.5

Use components to determine the displacement of a cross-country skier who travelled 15.0 m [220°] and then 25.0 m [335°] (Figures 3.23 and 3.24).

Figure 3.23

Given
$d_1 = 15.0$ m [220°]
$d_2 = 25.0$ m [335°]

Required
displacement ($\vec{d}$)

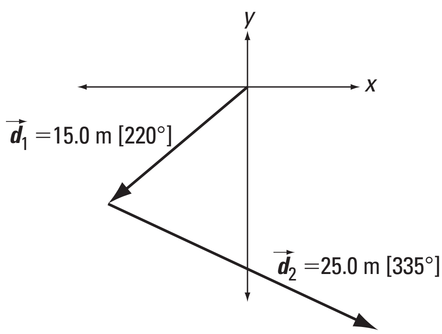


Figure 3.24

Analysis and Solution
Step 1: Use $d_x = d\cos\theta$ and $d_y = d\sin\theta$ to resolve each vector into its x and y components. Designate up and to the right as positive. Work with acute angles (Figure 3.25).

x direction:

$$\begin{aligned}d_{1_x} &= -(15.0\text{ m})(\cos 40°)\\ &= -11.49\text{ m}\end{aligned}$$

$$\begin{aligned}d_{2_x} &= (25.0\text{ m})(\cos 25°)\\ &= 22.66\text{ m}\end{aligned}$$

y direction:

$$\begin{aligned}d_{1_y} &= -(15.0\text{ m})(\sin 40°)\\ &= -9.642\text{ m}\end{aligned}$$

$$\begin{aligned}d_{2_y} &= -(25.0\text{ m})(\sin 25°)\\ &= -10.57\text{ m}\end{aligned}$$

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Explore More
How is the addition of more than two vectors accomplished?

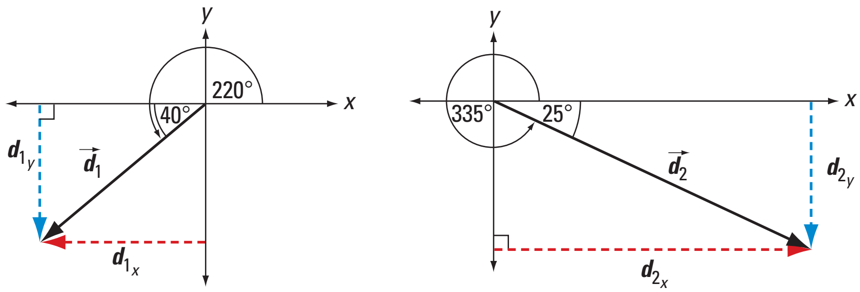

Figure 3.25

Step 2: Add the x and y components separately.

$$d_x = d_{1_x} + d_{2_x} = -11.49\text{ m} + 22.66\text{ m} = 11.17\text{ m}$$

$$d_y = d_{1_y} + d_{2_y} = -9.642\text{ m} + (-10.57\text{ m}) = -20.21\text{ m}$$

Step 3: To find the magnitude of the resultant, calculate d using the Pythagorean theorem (Figure 3.26).

$$d^2 = (d_x)^2 + (d_y)^2 = (11.17\text{ m})^2 + (-20.21\text{ m})^2$$

$$d = \sqrt{(11.17\text{ m})^2 + (-20.21\text{ m})^2} = 23.09\text{ m}$$

Figure 3.27 shows that the resultant lies below the positive x-axis.

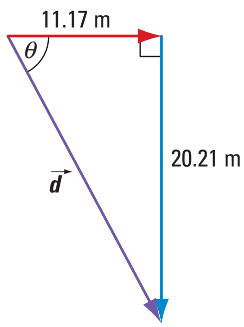

Figure 3.26

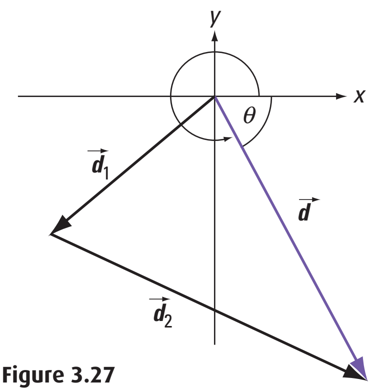

Figure 3.27

Step 4: To find the angle, use the tangent function (Figure 3.26).

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{20.21\text{ m}}{11.17\text{ m}} = 1.810$$

$$\theta = \tan^{-1}(1.810) = 61°$$

From Figure 3.27, note that the angle, θ, lies below the positive x-axis. If the angle is measured counterclockwise from the positive x-axis it is $360° - 61° = 299°$.

Paraphrase

The cross-country skier's displacement is 23.1 m [299°].

Practice Problems

1. Find the displacement of a farmer who walked 80.0 m [0°] and then 60.0 m [335°].
2. Find the displacement of a soccer player who runs 15 m [15° N of E] and then 13 m [5° W of N].
3. While tracking a bear, a wildlife biologist travels 300 m [S] and then 550 m [75° N of E]. What is her displacement?

Answers

1. 137 m [349°]
2. 21 m [52° N of E]
3. 272 m [58° N of E]

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Take It Further

Pilots use radar vectors when landing their aircraft. Radar vectors are issued by air traffic control to fly in a particular direction and usually include altitude and speed restrictions. Research the effect of winds on radar vectors. Write a brief summary of your findings.

In summary, in order to solve a two-dimensional motion problem, you need to:

1. Split the motion into two one-dimensional problems by using the vectors' x and y components.
2. Then add the x and y components separately.
3. Find the magnitude of the resultant using the Pythagorean theorem.
4. Find the angle of the resultant using the tangent function.

A11 Just-in-Time Math

PHYSICS•SOURCE

Diagonal of a Rectangle

Activity Overview

In this activity, you will review a method to calculate the diagonal of a rectangle. This method is useful in calculating the resultant vector when the two component vectors form a right triangle with the resultant vector.

Your teacher will give you a full copy of the activity.

A12 Quick Lab

PHYSICS•SOURCE

Vector Walk

Purpose

To compare the endpoint and displacement when vectors are added in a different order.

Activity Overview

In this activity, you and your partner will start at the centre line of a football field and walk along four vectors (Figure 3.28). You will compare your endpoint and displacement with other groups who walked the vectors in a different order.

Your teacher will give you a copy of the full activity.

Prelab Questions

Consider the questions below before beginning this activity.

1. Describe how you get to the gym from your physics classroom using
   (a) compass directions
   (b) the words: right, left, and straight
2. Is the magnitude of the resultant vector always less than the sum of the magnitudes of two non-collinear vectors? Explain.

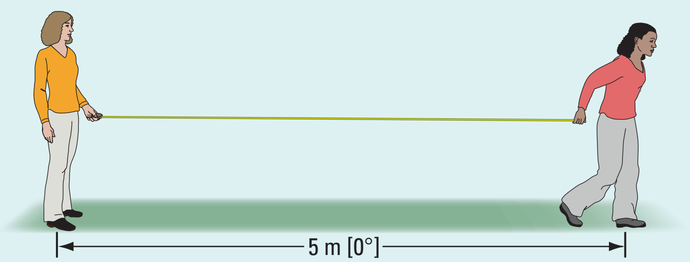

Figure 3.28 The vector walk

3.2 Check and Reflect

Key Concept Review

1. What trigonometric ratios can be used to determine the x or horizontal component of a vector? Draw diagrams to illustrate your answers.
2. Are the following statements true or false? Justify your answer.
   (a) The order in which vectors are added is important.
   (b) Displacement and distance are always equal.
3. Find v_x and v_y for the following vectors:
   (a) A boat travelling at 15 km/h [45° N of W]
   (b) A plane flying at 200 km/h [25° E of S]
   (c) A mountain bike travelling at 10 km/h [N]
4. Find the magnitude and direction of the vectors with the following components.
   (a) $d_x = 12$ m, $d_y = 7$ m
   (b) $v_x = 40$ km/h, $v_y = 55$ km/h
   (c) $d_x = 30$ cm, $d_y = -10$ cm

Connect Your Understanding

5. A camper left her tent to go to the lake. She walked 0.80 km [S], then 1.20 km [E], and 0.30 km [N].
   (a) Find her resultant displacement.
   (b) Add the vectors in two different orders and obtain the resultant for each case.
6. A student runs through a field 100 m [E], then 200 m [S], and finally 50 m [45° S of E]. Find his final position relative to his starting point.
7. A student has created a short computer program that calculates components of vectors drawn with a computer mouse. To demonstrate his program, he drags the mouse to create a vector at 55 cm [30° W of S]. What are the components of the vector?
8. Determine the distance travelled and the displacement for each of the following.
   (a) In-line skating through a park takes you 5.0 km [W], 3.0 km [N], 2.0 km [E], and 1.5 km [S].
   (b) A swimmer travels in a northerly direction across a 500-m-wide lake. Once across, the swimmer notices that she is 150 m east of her original starting position.
   (c) After leaving her cabin, a camper snowshoes 750 m [N] and then 2.20 km [S].
9. A boat sails 5.0 km [45° W of N]. It then changes direction and sails 7.0 km [45° S of E]. Where does the boat end up with reference to its starting point?
10. How much time can you save travelling diagonally instead of running 450 m [S] and then 650 m [W] if your running speed is 5.0 m/s?
11. A pellet gun fires a pellet with a velocity of 355 m/s at an angle of 30° to the horizontal. What is the magnitude of the vertical component of the velocity at the moment the pellet is fired?
12. Tourists on a jet ski move 1.20 km [55° N of E] and then 3.15 km [70° S of E]. Determine the jet ski's displacement.
13. A jogger runs with a velocity of 6.0 km/h [25° N of W] for 35 min and then changes direction, jogging for 20 min at 4.5 km/h [65° E of N]. Using a vector diagram, determine the jogger's total displacement and his average velocity for the workout.
14. Given that a baseball diamond is a square, assume that the first-base line is the horizontal axis. On second base, a baseball player's displacement from home plate is 38 m [45°].
   (a) What are the components of the player's displacement from home plate?
   (b) Has the runner standing on second base travelled a distance of 38 m? Why or why not?
15. Determine the resultant displacement of a skateboarder who rolls 45.0 m [310°] and 35.0 m [135°].

Reflection

16. Describe three examples of two dimensional motion you have experienced or observed.

For more questions, go to PHYSICS•SOURCE

3.3 Two Dimensional Projectiles

Section Summary

- g is a constant. The factors that affect the trajectory of a projectile are the initial velocity and the launch angle.
- The shape of the path of a projectile is parabolic.
- The total time of the flight for a projectile launched horizontally is the same as for free fall.
- At maximum height, a projectile's vertical velocity is zero.
- The higher the launch angle, the greater the maximum height of a projectile.

Sports are really science experiments in action. Consider golf balls, footballs, and tennis balls. All of these objects are projectiles (Figure 3.29). You know from personal experience that there is a relationship between the distance you can throw a ball and the angle at which it is thrown.

Figure 3.29 Sports is projectile motion in action.

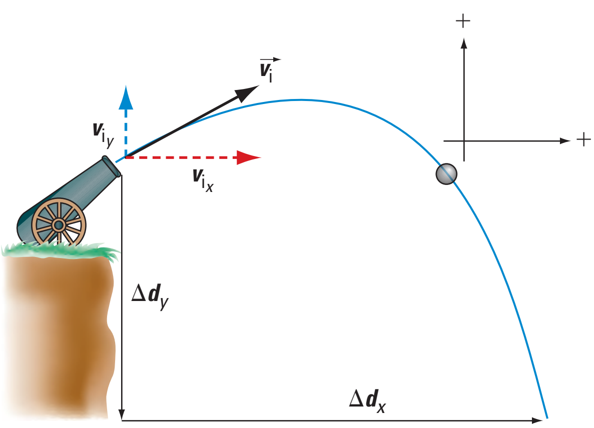

Figure 3.30 A projectile has a parabolic trajectory.

Galileo studied projectiles and found that they moved in two directions at the same time. He determined that the motion of a projectile, disregarding air resistance, follows the curved path of a parabola. The parabolic path of a projectile is called its **trajectory** (Figure 3.30). The shape of a projectile's trajectory depends on its initial velocity — both its initial speed and direction — and on acceleration due to gravity. To understand and analyze projectile motion, you need to consider the horizontal (x direction) and vertical (y direction) components of the object's motion separately.

A Projectile with Initial Horizontal Velocity

If you roll a ball on a table and it then falls off the edge of the table, its initial velocity is in the horizontal (x) direction only ($t = 0$). As the ball rolls off the table, it starts to accelerate in the y direction because acceleration due to gravity $a_y = g$ is acting vertically downwards. There is no acceleration in the x direction ($a_x = 0$). This tells us that the magnitude of the velocity in the x direction will remain a constant, but the magnitude of the velocity in the y direction will increase as shown in the Figure 3.31.

From Figure 3.32, note that gravity has no effect on an object's horizontal motion. So, the two components of a projectile's motion can be considered independently. As a result, a projectile experiences both uniform motion and uniformly accelerated motion at the same time. The horizontal motion of a projectile is an example of uniform motion; the projectile's horizontal velocity component is constant. The vertical motion of a projectile is an example of uniformly accelerated motion. The object's acceleration is the constant acceleration due to gravity or 9.81 m/s² [down] (neglecting friction).

PHYSICS • SOURCE

Suggested Activity

- A13 Quick Lab Overview on page 83

PHYSICS • SOURCE

Explore More

What is the effect of air resistance on projectile motion?

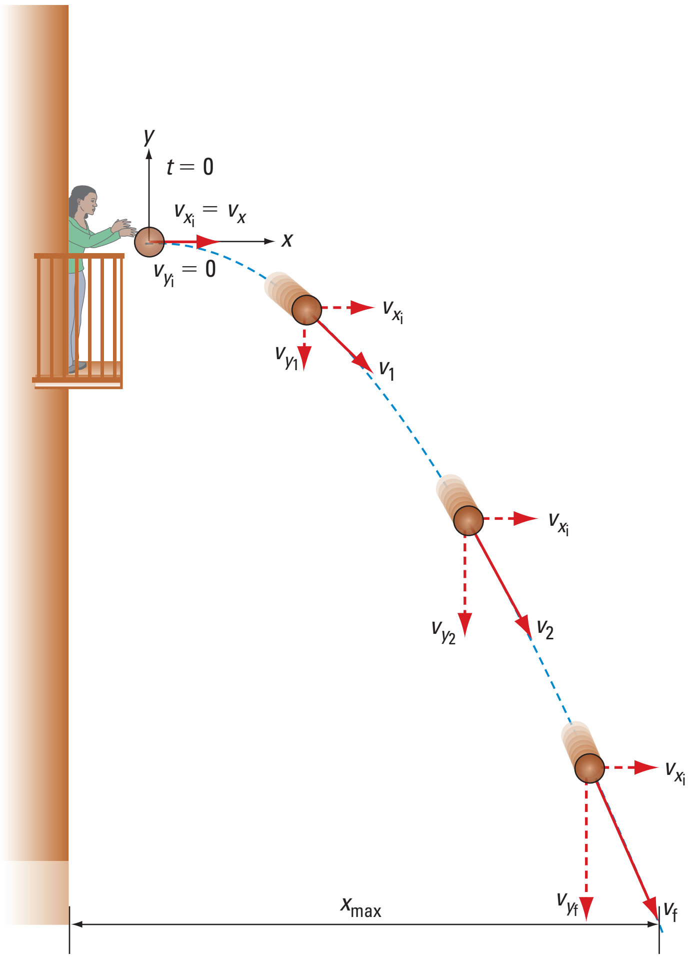

Figure 3.31 The velocity components of a projectile launched horizontally show that the projectile travels to the right as it falls downward.

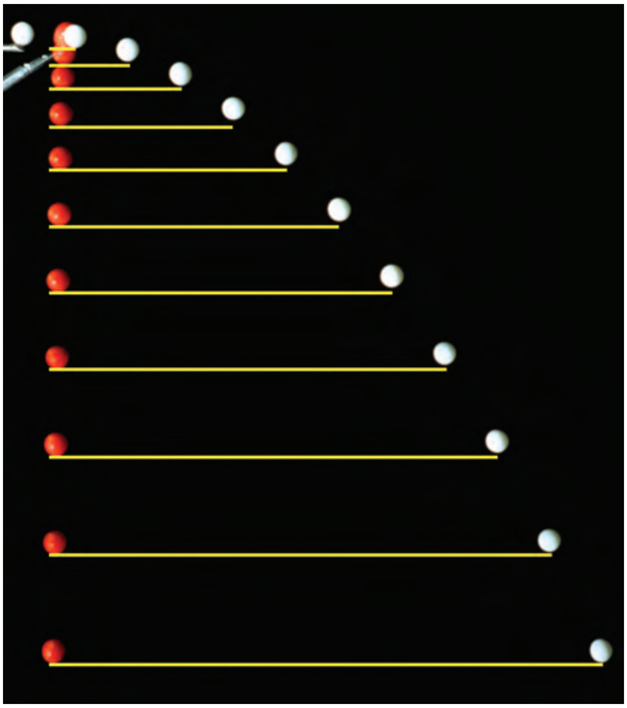

Figure 3.32 A multiflash photograph shows the paths of two golf balls. One was projected horizontally at the same time that the other was dropped straight down.

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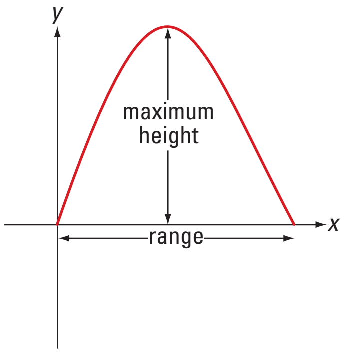

Figure 3.33 The range of a projectile is its horizontal distance travelled.

PHYSICS•SOURCE

Suggested Activity

- A14 Quick Lab Overview on page 83

Practice Problems

1. A coin rolls off a table with an initial horizontal speed of 30 cm/s. How far will the coin land from the base of the table if the table's height is 1.25 m?
2. An arrow is fired horizontally with a speed of 25.0 m/s from the top of a 150.0-m-tall cliff. Assuming no air resistance, determine the distance the arrow will drop in 2.50 s.
3. What is the horizontal speed of an object if it lands 40.0 m away from the base of a 100-m-tall cliff?

Answers

1. 15 cm
2. 30.7 m
3. 8.86 m/s

Refer to Figure 3.31. Since the projectile has a horizontal velocity component, it travels a horizontal distance along the ground from its initial launch point. This distance is called the projectile's **range** (Figure 3.33).

Example 3.6

An arrow is shot from a height of 20.0 m (Figure 3.34) with an initial horizontal velocity of 18.0 m/s. How far in the horizontal direction will the arrow fall to the ground?

Given

For convenience, choose forward and down to be positive because the motion is forward and down (Figure 3.35).

x direction

$v_{i_x} = 18.0\text{ m/s}$

y direction

$a_y = 9.81\text{ m/s}^2\text{ [down]} = +9.81\text{ m/s}^2$

$\Delta d_y = 20.0\text{ m}$

Figure 3.34

Required

distance from the base of the cliff (Δd_x)

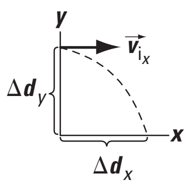

Figure 3.35

Analysis and Solution

Since there is no vertical component to the initial velocity of the arrow, $v_{i_y} = 0$ m/s. Therefore, the arrow experiences uniformly accelerated motion due to gravity in the vertical direction but uniform motion in the horizontal direction.

From the given values, note that, in the *y* direction, you have all the variables except for time. So, you can solve for time in the *y* direction, which is the time it takes the arrow to fall to the ground.

y direction:

$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$
$$= 0 + \frac{1}{2}a_y(\Delta t)^2$$
$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$
$$= \sqrt{\frac{2(20.0\text{ m})}{9.81\text{ m/s}^2}}$$
$$= 2.019\text{ s}$$

x direction:

The time taken for the arrow to fall vertically equals the time it travels horizontally. Substitute the value for time you found in the *y* direction to find the range. Since the arrow had a uniform horizontal speed of 18.0 m/s,

$$\Delta d_x = v_{i_x}\Delta t$$
$$= (18.0\text{ m/s})(2.019\text{ s})$$
$$= 36.3\text{ m}$$

Paraphrase

The arrow would land 36.3 m from the base of the cliff.

Objects Launched at an Angle to the Horizontal

Baseball is a projectile game (Figure 3.36). The pitcher throws a ball at the batter, who hits it to an open area in the field. The outfielder catches the ball and throws it to second base. The runner is out. All aspects of this sequence involve projectile motion. Each sequence requires a different angle on the throw and a different speed. If a player miscalculates one of these variables, the action fails: Pitchers throw wild pitches, batters strike out, and outfielders overthrow the bases. Winning the game depends on accurately predicting the components of the initial velocity of the ball.

Figure 3.36 Baseball is all about projectile motion

For objects launched at an angle, such as a baseball, the velocity of the object has both a horizontal and a vertical component. As we learned in section 3.2, any vector quantity can be resolved into x and y components using the trigonometric ratios

$d_x = d\cos\theta$ and $d_y = d\sin\theta$,

when θ is measured relative to the x-axis. To determine the horizontal and vertical components of velocity, this relationship becomes

$v_x = v\cos\theta$ and $v_y = v\sin\theta$,

as shown in Figure 3.37.

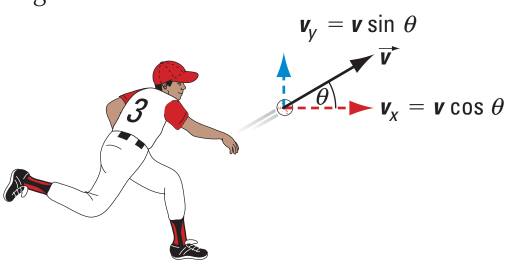

Figure 3.37 The horizontal and vertical components of velocity

Solving problems involving objects launched at an angle is similar to solving problems involving objects launched horizontally. However, in this case, the initial angle θ_i is not zero.

Refer to Figure 3.38 on the next page to understand the change in the variables with respect to each other. Also, observe that the y-component of the velocity, v_y, at the maximum height is zero.

Actually, the maximum height, *h,* is reached when this component reduces to zero because of acceleration due to gravity. The velocity at this point has a horizontal direction, which is the *x*-component of the initial velocity. At this point, the acceleration vector (pointing vertically downwards) and the velocity vector are perpendicular to each other. The projectile continues to move forward as shown in Figure 3.38 because of these two vectors, and the shape of the projectile will be parabolic. If the velocity vector did not have this horizontal component, the object would have fallen vertically downwards due to the acceleration vector.

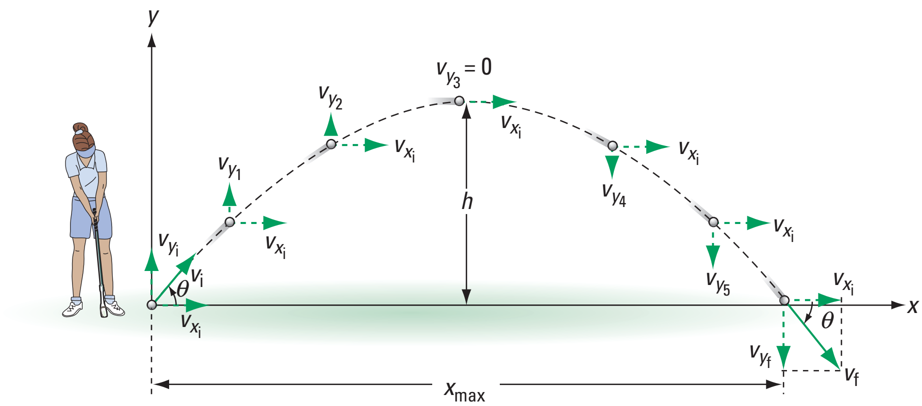

Figure 3.38 The velocity components of the ball are shown for various times.

The total time of flight for the projectile is twice the time it takes to reach the maximum height. The range is the total displacement in the *x* direction during this total time. The maximum height of the projectile $\left(h = \dfrac{{v_{i_y}}^2}{2g}\right)$ depends only on the *y*-component ($v_{i_y} = v_i \sin\theta_i$) of the initial velocity, as *g* is a constant. Moreover, as θ increases, sin θ also increases, which means that as the initial or launch angle increases, the maximum height increases (Figure 3.39). If a basketball player or high jumper wants to jump higher, he should keep the initial angle as close to 90° as possible.

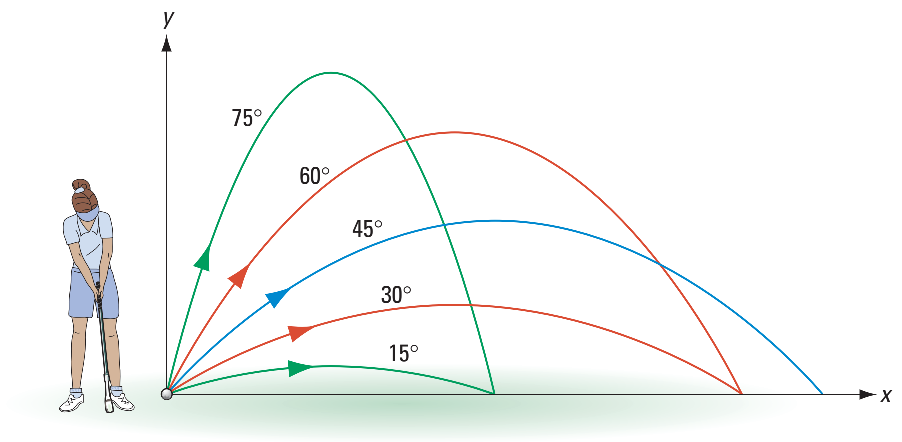

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Figure 3.39 The maximum height of a projectile increases as the launch angle increases.

Example 3.7

Baseball players often practise their swing in a batting cage, in which a pitching machine delivers the ball (Figure 3.40). If the baseball is launched with an initial velocity of 22.0 m/s [30.0°] and the player hits it at the same height from which it was launched, how long is the baseball in the air on its way to the batter?

Figure 3.40

Given

$\vec{v}_i = 22$ m/s [30.0°]

Required

time (Δt)

Analysis and Solution

Choose forward and up to be positive (Figure 3.41). First find the components of the baseball's initial velocity.

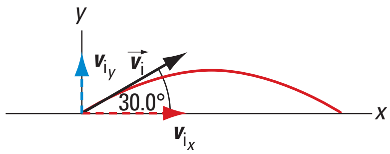

Figure 3.41

x direction

$$\begin{aligned} v_{i_x} &= v_i \cos\theta \\ &= (22.0 \text{ m/s})(\cos 30.0°) \\ &= 19.05 \text{ m/s} \end{aligned}$$

y direction

$$\begin{aligned} v_{i_y} &= v_i \sin\theta \\ &= (22.0 \text{ m/s})(\sin 30.0°) \\ &= 11.00 \text{ m/s} \end{aligned}$$

Since the ball returns to the same height from which it was launched, $\Delta d_y = 0$. With this extra known quantity, you now have enough information in the y direction to find the time the ball spent in the air.

$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$0 = (11.00 \text{ m/s})\Delta t + \frac{1}{2}(-9.81 \text{ m/s}^2)(\Delta t)^2$$

Isolate Δt and solve.

$$(4.905 \text{ m/s}^2)(\Delta t)^{\cancel{2}} = (11.00 \text{ m/s})(\cancel{\Delta t})$$

$$\begin{aligned} \Delta t &= \frac{11.00 \cancel{\text{m}}/\text{s}}{4.905 \cancel{\text{m}}/\text{s}^{\cancel{2}}} \\ &= 2.24 \text{ s} \end{aligned}$$

Paraphrase

The baseball is in the air for 2.24 s.

Practice Problems

1. A ball thrown horizontally at 10.0 m/s travels for 3.0 s before it strikes the ground. Find
   (a) the distance it travels horizontally
   (b) the height from which it was thrown
2. A ball is thrown with a velocity of 20.0 m/s [30°] and travels for 3.0 s before it strikes the ground. Find
   (a) the distance it travels horizontally
   (b) the height from which it was thrown.
   (c) the maximum height of the ball

Answers

1. (a) 30 m
   (b) 44 m
2. (a) 52 m
   (b) 14 m
   (c) 19 m

In Example 3.7, what is the horizontal distance from where the ball is launched to where the ball is hit? Since horizontal velocity is constant,

$$\begin{aligned} \Delta d_x &= v_{i_x}\Delta t \\ &= (19.05 \text{ m/s})(2.24 \text{ s}) \\ &= 42.7 \text{ m} \end{aligned}$$

The baseball would travel a horizontal distance of 42.7 m.

In the next example, you are given the time and are asked to solve for one of the other variables. However, the style of solving the problem remains the same. In any problem that you will be asked to solve in this course, you will always be able to solve for one quantity in either the *x* or *y* direction, and then you can substitute your answer to solve for the remaining variable(s).

Example 3.8

A paintball directed at a target is shot at an angle of 25.0° (Figure 3.42). If paint splats on its intended target at the same height from which it was launched, 3.00 s later, find the distance from the shooter to the target.

Given

Choose down and right to be positive.

$\vec{a} = a_y = 9.81\ \text{m/s}^2\ [\text{down}] = +9.81\ \text{m/s}^2$

$\theta = 25.0°$

$\Delta t = 3.00\ \text{s}$

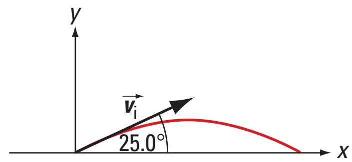

Figure 3.42

Required

range (Δd_x)

Analysis and Solution

Use the equation $\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$. Since the height of landing is the same as the launch height, $\Delta d_y = 0$.

y direction:

$$\Delta d_y = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$0 = v_{i_y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$v_{i_y}\Delta t = -\frac{1}{2}a_y(\Delta t)^2$$

$$\begin{aligned} v_{i_y} &= -\frac{1}{2}a_y\Delta t \\ &= -\frac{1}{2}\left(9.81\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(3.00\ \cancel{\text{s}}) \\ &= -14.7\ \text{m/s} \end{aligned}$$

Since down is positive, the negative sign means that the direction of the vertical component of initial velocity is up.

x direction:

Find the initial horizontal speed using the tangent function. Because there is no acceleration in the *x* direction, the ball's horizontal speed remains the same during its flight: $a_x = 0$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\begin{aligned} \text{adjacent} &= \frac{\text{opposite}}{\tan\theta} \\ &= \frac{14.7\ \text{m/s}}{\tan 25.0°} \\ &= 31.56\ \text{m/s} \end{aligned}$$

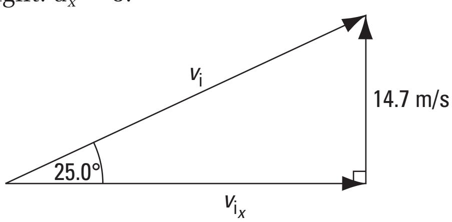

Figure 3.43

From Figure 3.43, the adjacent side is v_{i_x} and it points to the right, so $v_{i_x} = 31.56$ m/s.

Practice Problems

1. Determine the height reached by a baseball if it is released with a velocity of 17.0 m/s [20°].
2. A German U2 rocket from the Second World War had a range of 300 km, reaching a maximum height of 100 km. Determine the rocket's initial velocity.

Answers

1. 1.72 m
2. 1.75×10^3 m/s [53.1°]

Now find the horizontal distance travelled.

$\Delta d_x = v_{i_x} \Delta t$
$= (31.56 \text{ m/s})(3.00 \text{ s})$
$= 94.7 \text{ m}$

Paraphrase

The distance that separates the target from the shooter is 94.7 m.

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Take It Further

Jai-alai once held the record for the fastest speed for a projectile in any ball game. It was 302 km/h. Research the sport of jai-alai. Why are the balls able to reach such high speeds?

A13 Quick Lab

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Which Lands First?

Purpose

To observe the relationship between horizontal and vertical motion of objects on a ramp

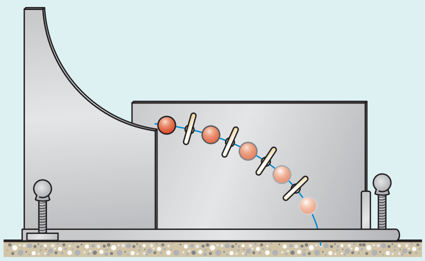

Figure 3.44 Galileo apparatus

Activity Overview

In this activity, you will use the Galileo apparatus (Figure 3.44) and steel balls released from the top of each ramp to determine which ball hits the ground first. You will repeat this for different ramps.

Your teacher will give you a copy of the full activity.

Prelab Questions

Consider the questions below before beginning this activity.

1. A pencil and a science textbook are dropped at the same time from the same height. Explain why they land on the floor at the same time.

2. What factors would affect the result in Question 1?

DI Key Activity

A14 Quick Lab

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Shuffleboard Darts

Purpose

To observe how the initial horizontal velocity affects the range of a horizontal projectile

Activity Overview

In this activity, you set up the activity as shown in Figure 3.45. By sliding or rolling objects of different size and mass, you can observe how the initial horizontal velocity affects the range of the projectile.

Your teacher will give you a copy of the full activity.

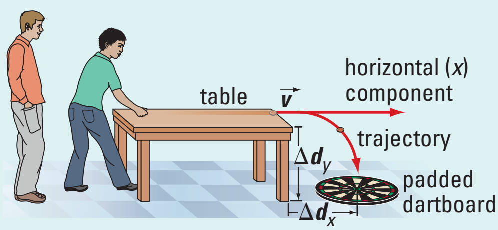


Figure 3.45 Setup for the activity

Prelab Questions

Consider the questions below before beginning this activity.

1. A snowball is thrown horizontally. What type of motion is the snowball experiencing vertically? Explain.

2. In Question 1, what type of motion is the snowball experiencing horizontally? Explain.

3.3 Check and Reflect

Key Concept Review

1. Platform divers receive lower marks if they enter the water a distance away from the platform, whereas speed swimmers dive as far out into the pool as they can. Compare and contrast the horizontal and vertical components of each type of athlete's motion.

2. For a fixed speed, how does the range depend on the angle, θ?

3. (a) For a horizontally fired projectile, is there a location on its trajectory where the acceleration and velocity vectors are perpendicular? Explain.
   (b) For a projectile, is there a location on its trajectory where the acceleration and velocity vectors are parallel? Explain.

4. Water safety instructors tell novice swimmers to put their toes over the edge and jump out into the pool. Explain why, using concepts from kinematics and projectile motion.

5. Classify the following as uniform or non-uniform motion for a horizontally launched projectile.
   (a) horizontal component of position
   (b) vertical component of position
   (c) horizontal component of velocity
   (d) vertical component of velocity
   (e) horizontal component of acceleration
   (f) vertical component of acceleration

Connect Your Understanding

6. Participants in a road race take water from a refreshment station and throw their empty cups away farther down the course. If a runner has a forward speed of 6.20 m/s, how far in advance of a garbage pail should he release his water cup if the vertical distance between the lid of the garbage can and the runner's point of release is 0.50 m?

7. A baseball is thrown with a velocity of 27.0 m/s [35°]. What are the components of the ball's initial velocity? How high and how far will it travel?

8. A football is thrown to a moving receiver. The football leaves the quarterback's hands 1.75 m above the ground with a velocity of 17.0 m/s [25°]. If the receiver starts 12.0 m away from the quarterback along the line of flight of the ball when it is thrown, what constant velocity must she have to get to the ball at the instant it is 1.75 m above the ground?

9. At the 2004 Olympic Games in Athens, Dwight Phillips won the gold medal in men's long jump with a jump of 8.59 m. If the angle of his jump was 23°, what was his takeoff speed? (Treat the jumper as an object; ignore that his legs are in front of him when he lands.)

10. A projectile is fired with an initial speed of 120 m/s at an angle of 55.0° above the horizontal from the top of a cliff 50.0 m high. Find
    (a) the time taken to reach maximum height
    (b) the maximum height with respect to the ground next to the cliff
    (c) the total time in the air
    (d) the range
    (e) the components of the final velocity just before the projectile hits the ground

11. Design a spreadsheet to determine the maximum height and range of a projectile with a launch angle that increases from 0° to 90° and whose initial speed is 20.0 m/s.

12. During the Apollo 14 mission, Alan Shepard was the first person to hit a golf ball on the Moon. If the golf ball was launched from the Moon's surface with a velocity of 50 m/s [35°] and the acceleration due to gravity on the Moon is -1.61 m/s^2,
    (a) how long was the golf ball in the air?
    (b) what was the golf ball's range?

Reflection

13. (a) What do you think is the most interesting information you learned in this section?
    (b) How does this information connect with what you already knew about the subject?

For more questions, go to PHYSICS•SOURCE

Great CANADIANS in Physics — Bertram Neville Brockhouse

Figure 3.46 Dr. Bertram Brockhouse

Bertram Neville Brockhouse was born in 1918 in Lethbridge, Alberta, Canada. He was the co-recipient of the Nobel Prize in Physics in 1994 for his work on the investigation of solids and liquids using neutron scattering. The method used by him has proved to be very useful in the study of the structure and dynamics of solids, liquids, and crystals. Neutrons are electrically neutral and when moving neutrons are used to bombard an atom's nucleus they are not affected by the electrons. When neutrons collide with nuclei in crystals, the analysis of their motion reveals very important properties of the material. Because neutrons are heavier than electrons, they interact with light elements and their motion is used to identify isotopes.

From the time Dr. Brockhouse was young, he had a passion for mathematics and physics. After completing his undergraduate degree with first class honours from the University of British Columbia, he received his PhD in physics in 1950 from the University of Toronto. He then was a professor in the department of physics at McMaster University. Dr. Brockhouse received several other awards, including the Tory Medal of the Royal Society of Canada and American Express's Buckley Prize. He was a Fellow of the Royal Societies of Canada and London. He was influential in encouraging many students in his field of research. Dr. Brockhouse passed away in 2003.

Physics CAREERS — 3-D Video Game Designer

When playing a video game or watching a 3-D movie, it often feels as if we are part of the scenes. The spectacular graphics and the moving objects, like cars and motorcycles, look very real. All this is possible because 3-D video game designers have studied physics or engineering at a university level and they understand the laws and principles of physics. The designers have created programs like "Physics engine" which are used to construct a mathematical model of Newton's Laws. Some use computer animation physics, "game physics" or "collision detection" to make the 3-D effects appear more real. As the desire for making graphics look closer to reality increases, the need for the graphics designers to understand 3-D physics has become important. The designer is able to simulate the true experience in a 3-D environment and make the graphics more realistic when they use their knowledge of mechanics.

To become a 3-D video game designer, one requires a very strong background in physics, computer graphics, and mathematics at the post-secondary level. Some universities offer specialist courses for creating graphics software. Graduates of such programs usually join the game software industry.

Figure 3.47 A person playing a skateboarding game

To find out more, visit PHYSICS•SOURCE

CHAPTER 3 CHAPTER REVIEW

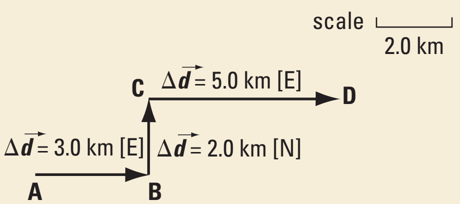
Key Concept Review

1. A juggler throws a ball vertically up in the air. Another person observes that the ball takes 4.75 s to go up and return to the juggler's hand. What was the maximum height reached by the ball above the juggler's hand? Assume there is no air resistance. k

2. An object takes 3.0 s to reach the maximum height when thrown straight up from a point. Assuming there is no air resistance, what was the initial velocity of this object? k

3. A child drops a small toy out of his window which is 9.0 m above the ground. What will be the velocity of the toy just before it hits the ground? k

4. During the Terry Fox Run, a participant starts from A, passes through B and C and travels to D. Using the information in the diagram below, find the participant's coordinates and displacement at B, C, and D from the initial point A (0.0 km, 0.0 km). Complete the table. k



Question 4

	Distance Δd (km)	Final Position	Displacement $\Delta\vec{d}$ (km) [direction]
A			
B			
C			
D			

5. Determine the x and y components of the displacement vector 55 m [222°]. k

6. What is the vertical component for velocity at the maximum height of a projectile's trajectory? k

7. During a field goal kick, as the football rises, what is the effect on the vertical component of its velocity? k

8. Fort McMurray is approximately 500 km [N] of Edmonton. Using a scale of 1.0 cm : 50.0 km, draw a displacement vector representing this distance. k

9. Give one reason why vector diagrams must be drawn to scale. k

10. Find the resultant for the following vectors:
(a) 5.0 m [S] and 10.0 m [N] k
(b) 65.0 cm [E] and 75.0 cm [E] k
(c) 1.0 km [forward] and 3.5 km [backward] k
(d) 35.0 km [right] − 45.0 km [left] k

11. For an object thrown vertically upward, what is the object's initial horizontal velocity? k

12. How much time can you save travelling diagonally instead of walking 750 m [N] and then 350 m [E] if you're walking speed is 1.5 m/s? t

13. How long will an arrow be in flight if it is shot at an angle of 25° and hits a target 50.0 m away, at the same elevation? a

Connect Your Understanding

14. A gymnast starts from rest and jumps vertically downwards. His velocity just before his feet touch the ground is 9.5 m/s in the same direction. From what height did the gymnast jump? t

15. The diving board at a swimming pool is at a height of 10.0 m above water. A diver dives vertically downwards in the pool. Assuming no air resistance, what will be her speed just before she hits the water surface? t

16. A ball is thrown up from a height of 1.5 m with a velocity of 5.0 m/s. How long will it take to fall to the ground? t

17. If a penny is dropped from the main deck of the CN Tower, its velocity just before it hits the ground below is almost 295 km/h. How high is the main deck (in metres)? a

18. In a circus tent, a trampoline is 2.0 m above the padded ground. After a clown jumps on the trampoline, he is thrown vertically up in the air with a velocity of 7.5 m/s. How high above the ground will the clown reach? t

ACHIEVEMENT CHART CATEGORIES

ⓚ Knowledge and understanding ⓣ Thinking and investigation
ⓒ Communication ⓐ Application

19. The driver of a bus travels 100 km west. She then drives 30 km north. Determine the resultant displacement of the bus. ⓐ

20. At what angle was an object thrown if its initial launch speed is 15.7 m/s, it remains airborne for 2.15 s, and travels 25.0 m horizontally? ⓣ

21. A coin rolls off a 25.0° incline on top of a 2.5-m-high bookcase with a speed of 30 m/s. How far from the base of the bookcase will the coin land? ⓣ

22. Starting from the left end of the hockey rink, the goal line is 3.96 m to the right of the boards, the blue line is 18.29 m to the right of the goal line, the next blue line is 16.46 m to the right of the first blue line, the far goal line is 18.29 m right, and the right boards are 3.96 m right of the goal line. How long is a standard NHL hockey rink? ⓣ

23. A ship sails 151 km [11° S of E]. It then changes course and sails 40 km [45° S of E]. Find the displacement of the ship. ⓐ

24. (a) How long will a soccer ball remain in flight if it is kicked with an initial velocity of 25.0 m/s [35.0°]? ⓐ
(b) How far down the field will the ball travel before it hits the ground? ⓐ
(c) What will be its maximum height? ⓐ

25. At what angle is an object launched if its initial vertical speed is 3.75 m/s and its initial horizontal speed is 4.50 m/s? ⓣ

26. A golf ball is hit with an initial velocity of 30.0 m/s [55°]. What are the ball's range and maximum height? ⓐ

27. Off the tee, a professional golfer can drive a ball with a velocity of 80.0 m/s [10°].
(a) How far will the ball travel horizontally before it hits the ground? ⓐ
(b) For how long is the ball airborne? ⓐ

28. Two boys are playing with a ball in their backyard. One of them throws the ball with a speed of 10.0 m/s at an angle of 37°. How far should the other boy stand so that he is able to catch the ball at the same height as the initial throw? ⓐ

29. An object is thrown horizontally off a cliff with an initial speed of 7.50 m/s. The object strikes the ground 3.0 s later. Find
(a) the object's vertical velocity component when it reaches the ground ⓐ
(b) the distance between the base of the cliff and the object when it strikes the ground ⓐ
(c) the horizontal velocity of the object 1.50 s after its release ⓐ

30. If a high jumper reaches her maximum height as she travels across the bar, determine the initial velocity she must have to clear a bar set at 2.0 m if her range during the jump is 2.0 m. What assumptions did you make to complete the calculations? ⓣ

31. The maximum speed a long jumper can attain is 7.0 m/s.
(a) What is the maximum range of the jumper? ⓐ
(b) How long does she remain in the air during the jump? ⓐ

Reflection

32. Write a paragraph describing the similarities and differences between motion in a horizontal plane and motion in a vertical plane. Share your thoughts with another classmate. ⓒ

33. Do you think that the knowledge about projectiles will help you in sports? Why? ⓒ

Unit Task Link

In the city of Toronto, red light cameras have been installed at major intersections. The purpose of the program is to improve safety by reducing red light running. How does the program work? Has there been an improvement in safety at these intersections?

For more questions, go to PHYSICS•SOURCE