ANSWERS - p. 440-441, # 34-67

25. $\angle Y \cong \angle W$
$m\angle Y = m\angle W = 54°$

26. $\angle Z \cong \angle X$
$m\angle Z = m\angle X = 126°$

27. Slope from R to S is rise of 2 and run of 10; rise of 2 from V to T is $-7 + 2 = -5$; run of 10 from V to T is $-4 + 10 = 6$; $T = (6, -5)$

28.

Statements	Reasons
1. $GHLM$ is a ▱. $\angle L \cong \angle JMG$	1. Given
2. $\angle G \cong \angle L$	2. ▱ → opp. ∠s ≅
3. $\angle G \cong \angle JMG$	3. Trans. Prop. of ≅
4. $\overline{GJ} \cong \overline{MJ}$	4. Conv. Isosc. △ Thm.
5. $\triangle GJM$ is isosc.	5. Def. of isosc. △

LESSON 6-3

29. $m = 13 \rightarrow m\angle G = 9(13) = 117°$; $n = 27 \rightarrow m\angle A = 2(27) + 9 = 63°$, $m\angle E = 3(27) - 18 = 63°$
Since $117° + 63° = 180°$ $\angle G$ is supp. to $\angle A$ and $\angle E$, so one ∠ of $ACEG$ is supp. to both of its cons. ∠s. $ACEG$ is a ▱ by Thm. 6-3-4.

30. $x = 25 \rightarrow m\angle Q = 4(25) + 4 = 104°$, $m\angle R = 3(25) + 1 = 76°$; so $\angle Q$ and $\angle R$ are supp. $y = 7 \rightarrow QT = 2(7) + 11 = 25$, $RS = 5(7) - 10 = 25$
By Conv. of Same-Side Int. ∠s Thm., $\overline{QT} \parallel \overline{RS}$; since $\overline{QT} \cong \overline{RS}$, $QRST$ is a ▱ by Thm. 6-3-1.

31. Yes; The diags. bisect each other. By Thm. 6-3-5 the quad. is a ▱.

Review 4-6.6

32. No; By Conv. of Alt. Int. ∠s Thm., one pair of opp. sides is ∥, but other pair is ≅. None of conditions for a ▱ are met.

33. slope of $\overline{BD} = \frac{2}{10} = \frac{1}{5}$; slope of $\overline{FH} = \frac{-2}{-10} = \frac{1}{5}$
slope of $\overline{BH} = \frac{-6}{1} = -6$; slope of $\overline{DF} = \frac{-6}{1} = -6$
Both pairs of opp. sides have the same slope, so $\overline{BD} \parallel \overline{FH}$ and $\overline{BH} \parallel \overline{DF}$; by def., $BDFH$ is a ▱.

LESSON 6-4

34. $\overline{AB} \cong \overline{CD}$
$AB = CD = 18$

35. $AC = 2CE$
$= 2(19.8) = 39.6$

36. $\overline{BD} \cong \overline{AC}$
$BD = AC = 39.6$

37. $\overline{BE} \cong \overline{CE}$
$BE = CE = 19.8$

38. $WX = WZ$
$7a + 1 = 9a - 6$
$7 = 2a$
$3.5 = a$
$WX = 7(3.5) + 1 = 25.5$

39. $\overline{XV} \cong \overline{VZ}$
$XV = VZ$
$= 3(3.5) = 10.5$

40. $\overline{XY} \cong \overline{WX}$
$XY = WX = 25.5$

41. $XZ = 2XV$
$= 2(10.5) = 21$

42. $m\angle TZV = 90$
$8n + 18 = 90$
$8n = 72$
$n = 9$
Think: $\overrightarrow{RT}$ bisects $\angle SRV$.
$m\angle TRS = \frac{1}{2} m\angle SRV$
$= \frac{1}{2}(9(9) + 1) = 41°$

43. $m\angle RSV + m\angle TRS = 90$
$m\angle RSV + 41 = 90$
$m\angle RSV = 49°$

44. $\angle STV \cong \angle SRV$
$m\angle STV = m\angle SRV$
$= 9(9) + 1 = 82°$

45. $m\angle TVR + m\angle STV = 180$
$m\angle TVR + 82 = 180$
$m\angle TVR = 98°$

46. Think: All 4 △s are isosc. △s.
$2m\angle 1 + m\angle 2 = 180$
$2m\angle 3 + m\angle 4 = 180$
By Lin. Pair Thm.,
$m\angle 2 + m\angle 4 = 180$
By Alt. Int ∠s Thm.,
$m\angle 3 = 33°$
$2(33) + m\angle 4 = 180$
$m\angle 4 = 114°$
$m\angle 2 + 114 = 180$
$m\angle 2 = 66°$
$2m\angle 1 + 66 = 180$
$2m\angle 1 = 114$
$m\angle 1 = 57°$
By Alt. Int ∠s Thm.,
$\angle 1 \cong \angle 5$
$m\angle 5 = m\angle 1 = 57°$

47. Think: All 4 △s are ≅ rt. △s.
$m\angle 2 = m\angle 5 = 53°$
$m\angle 3 = 90°$
$m\angle 4 + m\angle 5 = 90$
$m\angle 4 + 53 = 90$
$m\angle 4 = 37°$
$\angle 1 \cong \angle 4$
$m\angle 1 = m\angle 4 = 37°$

48. **Step 1** Show that $\overline{RT}$ and $\overline{SU}$ are congruent.
$RT = \sqrt{((-3) - (-5))^2 + (-6 - 0)^2} = 2\sqrt{10}$
$SU = \sqrt{((-7) - (-1))^2 + ((-4) - (-2))^2} = 2\sqrt{10}$
Since $RT = SU$, $\overline{RT} \cong \overline{SU}$

Step 2 Show that $\overline{RT}$ and $\overline{SU}$ are perpendicular.
slope of RT: $\frac{-6 - 0}{-3 - (-5)} = -3$
slope of SU: $\frac{-4 - (-2)}{-7 - (-1)} = \frac{1}{3}$
since $-3\left(\frac{1}{3}\right) = -1$, $\overline{RT} \perp \overline{SU}$

Step 3 Show that $\overline{RT}$ and $\overline{SU}$ bisect each other.
mdpt. of RT: $\left(\frac{-5 + (-3)}{2}, \frac{0 + (-6)}{2}\right) = (-4, -3)$
mdpt. of SU: $\left(\frac{-1 + (-7)}{2}, \frac{-2 + (-4)}{2}\right) = (-4, -3)$
Since $\overline{RT}$ and $\overline{SU}$ have the same midpoint, they bisect each other. The diagonals are congruent perpendicular biectors of each other.

49. Step 1 Show that $\overline{EG}$ and $\overline{FH}$ are congruent.

$EG = \sqrt{(5-2)^2 + (-2-1)^2} = 3\sqrt{2}$

$FH = \sqrt{(2-5)^2 + (-2-1)^2} = 3\sqrt{2}$

Since $EG = FH$, $\overline{EG} \cong \overline{FH}$

Step 2 Show that $\overline{EG}$ and $\overline{FH}$ are perpendicular.

slope of EG: $\frac{-2-1}{5-2} = -1$

slope of FH: $\frac{-2-1}{2-5} = 1$

since $-1(1) = -1$, $\overline{EG} \perp \overline{FH}$

Step 3 Show that $\overline{EG}$ and $\overline{FH}$ bisect each other.

mdpt. of RT: $\left(\frac{2+5}{2}, \frac{1+(-2)}{2}\right) = \left(\frac{7}{2}, -\frac{1}{2}\right)$

mdpt. of SU: $\left(\frac{5+2}{2}, \frac{1+(-2)}{2}\right) = \left(\frac{7}{2}, -\frac{1}{2}\right)$

Since $\overline{EG}$ and $\overline{FH}$ have the same midpoint, they bisect each other. The diagonals are congruent perpendicular biectors of each other.

LESSON 6-5

50. Not valid; by Thm. 6-5-2, if the diags. of a ▱ are ≅, then the ▱ is a rect. By Thm. 6-5-4, if the diags. of a ▱ are ⊥, then the ▱ is a rhombus. If a quad. is a rect. and a rhombus, then it is a square. But to apply this line of reasoning, you must first know that *EFRS* is a ▱.

51. valid (diags. bisect each other → ▱; ▱ with diags. ≅ → rect.)

52. valid (*EFRS* is a ▱ by def.; ▱ with 1 pair cons. sides ≅ → rhombus)

53. $BJ = \sqrt{8^2 + 8^2} = 8\sqrt{2}$; $FN = \sqrt{6^2 + 6^2} = 6\sqrt{2}$

Diags. are ≇, so ▱ is not a rect. Therefore ▱ is not a square.

slope of $\overline{BJ} = \frac{8}{8} = 1$; slope of $\overline{FN} = \frac{-6}{6} = -1$

Diags. are ⊥, so ▱ is a rhombus.

54. $DL = \sqrt{12^2 + 6^2} = 6\sqrt{5}$; $HP = \sqrt{6^2 + 12^2} = 6\sqrt{5}$

Diags. are ≅, so ▱ is a rect.

slope of $\overline{DL} = \frac{6}{12} = \frac{1}{2}$; slope of $\overline{HP} = \frac{-12}{-6} = 2$

Diags. are not ⊥, so ▱ is not a rhombus. Therefore ▱ is not a square.

55. $QW = \sqrt{12^2 + 8^2} = 4\sqrt{13}$; $TZ = \sqrt{8^2 + 12^2} = 4\sqrt{13}$

Diags. are ≅, so ▱ is a rect.

slope of $\overline{QW} = \frac{8}{12} = \frac{2}{3}$; slope of $\overline{TZ} = \frac{-12}{8} = -\frac{3}{2}$

Diags. are ⊥, so ▱ is a rhombus.

Rect., rhombus → ▱ is a square.

LESSON 6-6

56. Think: All 4 △ are rt. △; left pair of △ is ≅, as is right pair.

$m\angle XYZ = 2m\angle XYV$
$= 2(90 - m\angle VXY)$
$= 2(90 - 58)$
$= 64°$

57. $m\angle ZWV = \frac{1}{2}m\angle ZWX$
$= \frac{1}{2}(50) = 25°$

58. $m\angle VZW = 90 - m\angle ZWV$
$= 90 - 25 = 65°$

59. $m\angle WZY = m\angle VZW + m\angle VZY$
$= m\angle VZW + m\angle VXY$
$= 65 + 58 = 123°$

60. Think: Use Same-Side Int. ∠ Thm., isosc. trap. → base ∠ ≅.

$m\angle V + m\angle T = 180$
$m\angle V + 54 = 180$
$m\angle V = 126°$
$\angle R \cong \angle V$
$m\angle R = m\angle V = 126°$
$\angle S \cong \angle T$
$m\angle S = m\angle T = 54°$

61. Think: Isosc. trap. → diags. ≅

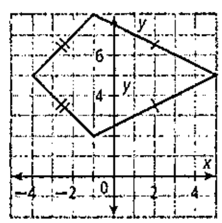
$\overline{BH} \cong \overline{EK}$
$BZ + ZH = EK$
$BZ + 70 = 121.6$
$BZ = 51.6$

62. $MN = \frac{1}{2}(AD + JG)$
$= \frac{1}{2}(67 + 30)$
$= 48.5$

63. $ST = \frac{1}{2}(FP + EQ)$
$2ST = FP + EQ$
$2(3.1) = 2.7 + EQ$
$3.5 = EQ$

64. $\angle P \cong \angle Y$
$m\angle P = m\angle Y$
$8n^2 - 11 = 6n^2 + 7$
$2n^2 = 18$
$n^2 = 9$
$n = \pm 3$

65.

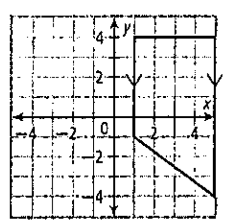


$AB = DA = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$BC = CD = \sqrt{6^2 + 3^2} = 3\sqrt{5}$

Exactly 2 pairs of cons. sides ≅ → kite.

66.

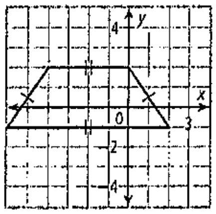


$\overline{AD}$ and $\overline{BC}$ are vert., so $\overline{AD} \parallel \overline{BC}$

$AB = |5 - 1| = 4$, $CD = \sqrt{4^2 + 3^2} = 5$

$\overline{AB} \ncong \overline{CD}$ and hence $\overline{AD} \nparallel \overline{BC}$

Exactly 1 pair opp. sides ∥, other pair ≇ → trap. (not isosc.)

67. 

$\overline{AD}$ and $\overline{BC}$ are horiz., but $AD = 2 + 6 = 8 \neq 4 = BC$
$\overline{AD} \parallel \overline{BC}$, $\overline{AD} \not\cong \overline{BC} \rightarrow$ trap. (not ▱)
$AB = \sqrt{2^2 + 3^2} = \sqrt{13}$; $CD = \sqrt{2^2 + 3^2} = \sqrt{13}$
$\overline{AB} \cong \overline{CD}$, not ▱ → isosc. trap

CHAPTER TEST, PAGE 442

1. not a polygon

2. polygon; decagon

3. Think: Use Quad. $\angle$ Sum Thm.
$$m\angle A + m\angle B + m\angle C + m\angle D = 360$$
$$12n + 14n + 8n + 11n = 360$$
$$45n = 360$$
$$n = 8$$
$m\angle A = 12(8) = 96°$, $m\angle B = 14(8) = 112°$,
$m\angle C = 8(8) = 64°$, $m\angle D = 11(8) = 88°$

4. $(9 - 2)180$
$(7)180$
$1260°$

5. $15\text{m(ext. } \angle) = 360$
$\text{m(ext. } \angle) = 24°$

6. Think: Z is mdpt. of $\overline{FH}$.
$FH = 2HZ$
$= 2(9) = 18$

Think: Cons. $\angle$s are supp.
$m\angle FEH + m\angle EHG = 180$
$m\angle FEH + 145 = 180$
$m\angle FEH = 35°$

7. Think: Opp. sides are ≅.
$JM = KL$
$4y - 7 = y + 11$
$3y = 18$
$y = 6$
$KL = 6 + 11 = 17$

Think: Cons. $\angle$s are supp.
$m\angle M + m\angle L = 180$
$6x - 1 + 2x + 9 = 180$
$8x = 172$
$x = 21.5$
$m\angle L = 2(21.5) + 9 = 52°$

8. Slope from S to R is rise of 4 and run of 1; from P to Q: rise of 4 is -3 to $-3 + 4 = 1$, run of 1 is -2 to $-2 + 1 = -1$; so coords. of $Q = (-1, 1)$.

9. $a = 4 \rightarrow XN = 3(4) = 12$, $NZ = 4 + 8 = 12$
$b = 3 \rightarrow WN = 4(3) + 3 = 15$, $NY = 5(3) = 15$
So N is mdpt. of $\overline{XZ}$ and $\overline{WY}$, and therefore diags. bisect each other. By Thm. 6-3-5, $WXYZ$ is a ▱.

10. No; one pair of opp. sides of the quad. are ∥. A pair of vert. $\angle$s formed by the diags. are ≅. None of conditions for a ▱ are met.

11. Possible answer: slope of $\overline{KL} = \frac{3}{9} = \frac{1}{3}$; slope of $\overline{ST} = \frac{-3}{-9} = \frac{1}{3}$
slope of $\overline{KT} = -\frac{4}{3}$; slope of $\overline{LS} = -\frac{4}{3}$
$\overline{KL} \parallel \overline{ST}$ and $\overline{KT} \parallel \overline{LS} \rightarrow KLST$ is a ▱.

12. $PT = \frac{1}{2}PC$
$= \frac{1}{2}LM$
$= \frac{1}{2}(23) = 11.5$

$\overline{PM} \cong \overline{LC}$
$PM = LC = 19$

13. Think: All 4 $\triangle$s are ≅ rt. $\triangle$s.
$m\angle NQK = 90$
$7z + 6 = 90$
$7z = 84$
$z = 12$
$m\angle HEQ = 90 - m\angle EHQ$
$= 90 - m\angle ENQ$
$= 90 - (5(12) + 1) = 29°$
$m\angle EHK = 2m\angle EHQ$
$= 2m\angle ENQ$
$= 2(5(12) + 1) = 122°$

14. Not valid; possible answer: $MNPQ$ is a rhombus by def. However, to show that $MNPQ$ is a square, you need to know that $MNPQ$ is also a rect.

15. valid ($MNPQ$ is a ▱ by def.; diags ≅ → $MNPQ$ is a rect.)

16. $AE = \sqrt{12^2 + 8^2} = 4\sqrt{13}$; $CG = \sqrt{4^2 + 6^2} = 2\sqrt{13}$
Diags. are ≇, so $ACEG$ is not a rect. Therefore $ACEG$ is not a square.
slope of $\overline{AE} = \frac{-8}{12} = -\frac{2}{3}$; slope of $\overline{CG} = \frac{-6}{-4} = \frac{3}{2}$
Diags. are ⊥, so $ACEG$ is a rhombus.

17. $PR = \sqrt{7^2 + 1^2} = \sqrt{50}$; $QS = \sqrt{5^2 + 5^2} = \sqrt{50}$
Diags. are ≅, so $PQRS$ is a rect.
slope of $\overline{PR} = \frac{1}{-7} = -\frac{1}{7}$; slope of $\overline{QS} = \frac{-5}{-5} = 1$
Diags. are not ⊥, so $PQRS$ is not a rhombus. Therefore $PQRS$ is not a square.

18. $m\angle FBN = m\angle FBR + m\angle RBN$
$= 90 - m\angle BFR + 90 - m\angle RNB$
$= 90 - m\angle JFR + 90 - \frac{1}{2}m\angle JNB$
$= 180 - 43 - \frac{1}{2}(68) = 103°$

19. $\overline{MS} \cong \overline{PV}$
$MY + YS = PV$
$MY + 24.7 = 61.1$
$MY = 36.4$

20. $XY = \frac{1}{2}(HR + GS)$
$2XY = HR + GS$
$2(25.5) = HR + 24$
$HR = 27$ in.