

## Exponential Equations Not Requiring Logarithms

Solve each equation.

1)  $4^{2x+3} = 1$

$4^{2x+3} = 4^0$

$2x+3=0$

$2x = -3$

$x = -3/2$

3)  $3^{1-2x} = 243$

$3^{1-2x} = 3^5$

$1-2x=5$

$-2x=4$

$x = -2$

5)  $4^{3x-2} = 1$

$4^{3x-2} = 4^0$

$3x-2=0$

$3x=2$

$x = 2/3$

7)  $6^{-2a} = 6^{2-3a}$

$-2a = 2-3a$

$a = 2$

9)  $6^{3m} \cdot 6^{-m} = 6^{-2m}$

$3m + -m = -2m$

$2m = -2m$

$4m = 0$

$m = 0$

11)  $10^{-3x} \cdot 10^x = \frac{1}{10}$

$10^{-3x} \cdot 10^x = 10^{-1}$

$-3x + x = -1$

$-2x = -1$

$x = 1/2$

2)  $5^{3-2x} = 5^{-x}$

$3-2x = -x$

$3 = x$

4)  $3^{2a} = 3^{-a}$

$2a = -a$

$3a = 0$

$a = 0$

6)  $4^{2p} = 4^{-2p-1}$

$2p = -2p-1$

$4p = -1$

$p = -1/4$

8)  $2^{2x+2} = 2^{3x}$

$2x+2 = 3x$

$2 = x$

10)  $\frac{2^x}{2^x} = 2^{-2x}$   $\rightarrow 2^x \cdot 2^{-x} = 2^{-2x}$  OR  $2^x = 2^{-2x} \cdot 2^x$

$1 = 2^{-2x}$

$x = -2x + x$

$2^0 = 2^{-2x}$

$x = -x$

$0 = -2x$

$2x = 0$

$x = 0$

12)  $3^{-2x+1} \cdot 3^{-2x-3} = 3^{-x}$

$-2x+1 + -2x-3 = -x$

$-4x-2 = -x$

$-3x = 2$

$x = -2/3$

$$13) 4^{-2x} \cdot 4^x = 64$$

$$4^{-2x} \cdot 4^x = 4^3$$

$$-2x + x = 3$$

$$-x = 3$$

$$x = -3$$

$$15) 2^x \cdot \frac{1}{32} = 32$$

$$2^x \cdot 2^{-5} = 2^5$$

$$x + -5 = 5$$

$$x = 10$$

$$17) 64 \cdot 16^{-3x} = 16^{3x-2}$$

$$4^3 \cdot (4^2)^{-3x} = (4^2)^{3x-2}$$

$$3 + -6x = 6x - 4$$

$$3 = 12x - 4$$

$$7 = 12x$$

$$x = 7/12$$

$$19) 81 \cdot 9^{-2b-2} = 27$$

$$3^4 \cdot (3^2)^{-2b-2} = 3^3$$

$$4 + -4b - 4 = 3$$

$$-4b = 3 \quad b = -3/4$$

$$21) \left(\frac{1}{6}\right)^{3x+2} \cdot 216^{3x} = \frac{1}{216}$$

$$(6^{-1})^{3x+2} \cdot (6^3)^{3x} = 6^{-3}$$

$$-3x - 2 + 9x = -3$$

$$+6x - 2 = -3$$

$$23) 16^r \cdot 64^{3-3r} = 64 \quad +6x = -1$$

$$(4^2)^r \cdot (4^3)^{3-3r} = 4^3 \quad x = 1/6$$

$$2r + 9 - 9r = 3$$

$$-7r + 9 = 3$$

$$-7r = -6$$

$$r = 6/7$$

$$14) 6^{-2x} \cdot 6^{-x} = \frac{1}{216}$$

$$6^{-2x} \cdot 6^{-x} = 6^{-3}$$

$$-2x - x = -3$$

$$-3x = -3$$

$$x = 1$$

$$16) 2^{-3p} \cdot 2^{2p} = 2^{2p}$$

$$-3p + 2p = 2p$$

$$-p = 2p$$

$$0 = 3p$$

$$p = 0$$

$$18) \frac{81^{3n+2}}{243^{-n}} = 3^4$$

$$81^{3n+2} = 3^4 \cdot 243^{-n}$$

$$(3^4)^{3n+2} = 3^4 \cdot (3^5)^{-n}$$

$$12n + 8 = 4 + -5n$$

$$17n = -4$$

$$20) 9^{-3x} \cdot 9^x = 27$$

$$(3^2)^{-3x} \cdot (3^2)^x = 3^3$$

$$-6x + 2x = 3$$

$$-4x = 3$$

$$x = -3/4$$

$$22) 243^{k+2} \cdot 9^{2k-1} = 9$$

$$(3^5)^{k+2} \cdot (3^2)^{2k-1} = 3^2$$

$$5k + 10 + 4k - 2 = 2$$

$$9k + 8 = 2$$

$$9k = -6$$

$$k = -6/9 = -2/3$$

$$24) 16^{2p-3} \cdot 4^{-2p} = 2^4$$

$$(2^4)^{2p-3} \cdot (2^2)^{-2p} = 2^4$$

$$8p - 12 + -4p = 4$$

$$4p - 12 = 4$$

$$4p = 16$$

$$p = 4$$

EXTRA PRACTICE 38  
Solving Exponential and Logarithmic Equations  
Use after Section 8.6

Name \_\_\_\_\_

KEY

Examples. Solve.

a)  $7^{x-1} = 343$   
 $7^{x-1} = 7^3$   
 $x-1 = 3$   
 $x = 4$

b)  $6^x = 15$   
 $\log 6^x = \log 15$   
 $x \log 6 = \log 15$   
 $x = \frac{\log 15}{\log 6}$   
 $x \approx \frac{1.1761}{0.7782}$   
 $x \approx 1.5113$

c)  $e^{-3t} = 0.04$   
 $\ln e^{-3t} = \ln 0.04$   
 $-3t \ln e = \ln 0.04$   
 $-3t = \ln 0.04$   
 $t = \frac{\ln 0.04}{-3}$   
 $t \approx \frac{-3.2189}{-3}$   
 $t \approx 1.073$

Solve.

1.  $3^{5x} = 81$   $x = 4/5$
2.  $e^{4t} = 120$   $t = \frac{\ln 120}{4}$
3.  $4^x = 6$   $x = \log_4 6 = \frac{\log 6}{\log 4}$
4.  $6^x = 2$   $x = \log_6 2 = \frac{\log 2}{\log 6}$
5.  $e^{-2t} = 0.6$   $t = \frac{\ln 0.6}{-2}$
6.  $5^{3x+2} = 625$   $x = 2/3$
7.  $8^{x+1} = 16$   $x = 1/3$
8.  $10^x = 7$   $x = \log 7$
9.  $7^x = 1520$   $x = \log_7 1520 = \frac{\log 1520}{\log 7}$
10.  $e^{0.04t} = 10$   $t = \frac{\ln 10}{0.04}$
11.  $e^{5t} = 5$   $t = \frac{\ln 5}{5}$
12.  $6^x = 7.1$   $\log_6 7.1 = \log 7.1 = x$
13.  $6^{x+3} = 36$   $x = -1$
14.  $4^{x-1} = 3$   $x = \log_4 3 + 1$
15.  $12^{2x-3} = 16$   $x = \frac{\log_2 16 + 3}{2}$
16.  $10^{5-x} = 1000$   $x = 2$

**EXTRA PRACTICE 38 (continued)**  
**Solving Exponential and Logarithmic Equations**  
 Use after Section 8.6

Example. Solve:  $\log_2(x+1) - \log_2(x-1) = 4$

Check:

$$\log_2(x+1) - \log_2(x-1) = 4$$

$$\log_2(x+1) - \log_2(x-1) = 4$$

$$\log_2 \frac{x+1}{x-1} = 4$$

$$\frac{x+1}{x-1} = 16$$

$$x+1 = 16x - 16$$

$$17 = 15x$$

$$\frac{17}{15} = x$$

The solution is  $\frac{17}{15}$ .

$$\begin{array}{l} \log_2 \left( \frac{17}{15} + 1 \right) - \log_2 \left( \frac{17}{15} - 1 \right) \\ \log_2 \frac{32}{15} - \log_2 \frac{2}{15} \\ \log_2 \left( \frac{32}{15} \div \frac{2}{15} \right) \\ \log_2 16 \\ 4 \end{array} \quad \Bigg| \quad 4$$

Solve.

17.  $\log x + \log(x+15) = 2$   
 $x = 5$

18.  $\log(x+2) - \log x = 3$   
 $x = 2.999$

19.  $\log_3(2x-7) = 4$   
 $x = 44$

20.  $\log_5(x-11) = 2$   
 $x = 36$

21.  $\log x + \log(x-21) = 2$   
 $x = 25$

22.  $\log_2(x-2) + \log_2(x+2) = 5$   
 $x = 6$

23.  $\log(3x+4) = 1$   
 $x = 2$

24.  $\log(x+33) - \log x = 2$   
 $x = 113$

25.  $\log x - \log(x+5) = -1$   
 $x = 5.9$

26.  $\log_4(x+3) - \log_4 x = 3$   
 $x = 21$

27.  $\log_4(x-6) + \log_4(x+6) = 3$   
 $x = 10$

28.  $\log_6 x + \log_6(x-9) = 2$   
 $x = 12$

29.  $\log x + \log(x-0.21) = -2$   
 $x = 114$

30.  $\log(x-48) + \log x = 2$   
 $x = 50$

31.  $\log_7 x + \log_7(4x+21) = 3$   
 $x = 7$

32.  $\log_2(5-x) = 4$   
 $x = -11$

## Logarithmic Equations

Solve each equation. (GIVE EXACT answers!)

1)  $\log(n+9) = \log 4n$

$$n+9 = 4n$$

$$9 = 3n$$

$$n = 3 \checkmark$$

2)  $\log(-5x) = \log(10-3x)$

$$-5x = 10 - 3x$$

$$-2x = 10$$

$$x = -5 \checkmark$$

3)  $\log(-3m-1) = \log(-4m-6)$

$$-3m-1 = -4m-6$$

$$m-1 = -6$$

$$m = -5 \checkmark$$

4)  $\log a = \log(4a-9)$

$$a = 4a - 9$$

$$-3a = -9$$

$$a = 3 \checkmark$$

5)  $-4\log_3 -9m = -4$

$$\log_3 -9m = 1$$

$$3^1 = -9m$$

$$3 = -9m$$

$$m = -1/3 \checkmark$$

6)  $7\log_9(x+8) = 7$

$$\log_9(x+8) = 1$$

$$9^1 = x+8$$

$$9 = x+8$$

$$x = 1 \checkmark$$

7)  $-8 + \log_9(m+1) = -8$

$$\log_9(m+1) = 0$$

$$9^0 = m+1$$

$$1 = m+1$$

$$m = 0 \checkmark$$

8)  $-2\log_8(a+1) = -8$

$$\log_8(a+1) = 4$$

$$8^4 = a+1$$

$$4096 = a+1$$

$$a = 4095 \checkmark$$

9)  $\log_2(a^2 - 6a) = \log_2(10 + 3a)$

$$a^2 - 6a = 10 + 3a$$

$$a^2 - 9a - 10 = 0$$

$$(a-10)(a+1) = 0$$

$$a = 10, a = -1 \checkmark$$

10)  $\log_{15}(x^2 + 13) = \log_{15}(-9x - 1)$

$$x^2 + 13 = -9x - 1$$

$$x^2 + 9x + 14 = 0$$

$$(x+2)(x+7) = 0$$

$$x = -2, x = -7 \checkmark$$

11)  $\log_{19}(x^2 + 17) = \log_{19}(8x + 2)$

$$x^2 + 17 = 8x + 2$$

$$x^2 - 8x + 15 = 0$$

$$(x-5)(x-3) = 0$$

$$x = 5, x = 3 \checkmark$$

12)  $\log_{12}(m^2 + 73) = \log_{12}(17m + 3)$

$$m^2 + 73 = 17m + 3$$

$$m^2 - 17m + 70 = 0$$

$$(m-7)(m-10) = 0$$

$$m = 7, m = 10 \checkmark$$

13)  $\log x - \log 6 = \log 15$

\*  $\log \frac{x}{6} = \log 15$

$\frac{x}{6} = 15$

$x = 90 \checkmark$

14)  $\log 7 + \log x = 2$

$\log 7x = 2$

$10^2 = 7x$

$100 = 7x$

$x = 100/7 \checkmark$

15)  $\log x + \log 2 = \log 2$

\*  $\log 2x = \log 2$

$2x = 2$

$x = 1 \checkmark$

16)  $\log x + \log 8 = 1$

$\log 8x = 1$

$10^1 = 8x$

$10 = 8x$

$x = 10/8 = 5/4 \checkmark$

17)  $\log_4 (x^2 - 3) + \log_4 10 = 1$

$\log_4 10(x^2 - 3) = 1$

$4^1 = 10x^2 - 30$

$0 = 10x^2 - 34$

$0 = 2(5x^2 - 17)$

$5x^2 - 17 = 0$

$5x^2 = 17$

$x^2 = \frac{17}{5}$

$x = \pm \sqrt{\frac{17}{5}}$

18)  $\log_7 2 + \log_7 (x - 5) = 2$

$\log_7 2(x - 5) = 2$

$\log_7 (2x - 10) = 2$

$7^2 = 2x - 10$

$49 = 2x - 10$

$59 = 2x$

$x = \frac{59}{2} \checkmark$

19)  $\log_5 3 - \log_5 5x = 2$

$\log_5 \frac{3}{5x} = 2$

$5^2 = \frac{3}{5x}$

$25 = \frac{3}{5x}$

$125x = 3$

$x = 3/125 \checkmark$

21)  $\ln(x+7) + \ln(x+3) = \ln 77$

$\ln(x+7)(x+3) = \ln 77$

$x^2 + 10x + 21 = 77$

\*  $x^2 + 10x - 56 = 0$

$(x+14)(x-4) = 0$

$x = -14, x = 4 \checkmark$

23)  $\ln(x+2) - \ln(x-1) = 1$

EXACT  $\ln \frac{x+2}{x-1} = 1$

$e = \frac{x+2}{x-1}$

$e(x-1) = x+2$

$ex - e = x+2$

$ex - x = e+2$

$x(e-1) = e+2$

$x = \frac{e+2}{e-1}$

EXACT \* 22)  $\ln(x+1) - \ln(x-1) = 3$

$\ln \frac{x+1}{x-1} = 3$

$e^3 = \frac{x+1}{x-1}$

$e^3(x-1) = x+1$

$e^3x - e^3 = x+1$

$e^3x - x = e^3 + 1$

$x(e^3 - 1) = e^3 + 1$

$x = \frac{e^3 + 1}{e^3 - 1}$

EXACT \* 24)  $\ln(x+3) - \ln(x+2) = 5$

$\ln \frac{x+3}{x+2} = 5$

$e^5 = \frac{x+3}{x+2}$

$e^5x + 2e^5 = x+3$

$e^5x - x = -2e^5 + 3$

$x(e^5 - 1) = -2e^5 + 3$

$x = \frac{-2e^5 + 3}{e^5 - 1}$