

- I. Divide the polynomials using polynomial long division.

1.  $(5x^3 - 13x^2 - x + 2) \div (x^2 - 3x + 1)$

$$\begin{array}{r} 5x+2 \\ x^2-3x+1 \overline{)5x^3-13x^2-x+2} \\ -(5x^3-15x^2+5x) \\ \hline 2x^2-6x+2 \\ -(2x^2-6x+2) \\ \hline 0 \end{array}$$

$$5x+2$$

2.  $(x^4 - 3x^2 + 2) \div (x^2 - 1)$

$$\begin{array}{r} x^2+0x-2 \\ x^2+0x-1 \overline{x^4+0x^3-3x^2+0x+2} \\ -(x^4+0x^3-x^2) \\ \hline 0x^3-2x^2+0x \\ -(0x^3+0x^2+0x) \\ \hline -2x^2+0x+2 \\ -(-2x^2+0x+2) \\ \hline 0 \end{array}$$

$$x^2-2$$

- II. Divide the polynomials using synthetic division.

1.  $(2x^3 + 2x^2 - x + 2) \div \left(x - \frac{1}{2}\right)$

$$\begin{array}{r} | 2 & 2 & -1 & 2 \\ 1 & & 1.5 & \frac{1}{4} \\ \hline 2 & 3 & \frac{1}{2} & \frac{9}{4} \end{array}$$

$$2x^3+3x+\frac{1}{2} + \frac{\frac{9}{4}}{(x-\frac{1}{2})}$$

2.  $(3x^3 - 10x^2 + 12x - 22) \div (x - 4)$

$$\begin{array}{r} | 3 & -10 & 12 & -22 \\ 4 & & 12 & 8 \\ \hline 3 & 2 & 20 & 58 \end{array}$$

$$3x^3+2x+20 + \frac{58}{(x-4)}$$

III. Use synthetic division to evaluate each function.

1. Given  $f(x) = 4x^3 - 13x + 10$ . Find  $f(8)$ .

$$\begin{array}{r} 8 | 4 \ 0 \ -13 \ 10 \\ \quad\quad\quad 32 \ 256 \ 1944 \\ \hline 4 \ 32 \ 243 \ 1954 \end{array}$$

$$f(8) = 1954$$

IV. Use synthetic division to show that the given binomial is a factor.

1.  $g(x) = x^4 - 11x^3 + 41x^2 - 61x + 30$  factor  $(x - 5)$

$$\begin{array}{r} 5 | 1 \ -11 \ 41 \ -61 \ 30 \\ \quad\quad\quad 5 \ -30 \ 55 \ -30 \\ \hline 1 \ -6 \ 11 \ -6 \ 0 \end{array}$$

It is a factor

← remainder is 0

V. Use the Rational Zero Test to determine the set of possible rational zeros.

1.  $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$

$$\frac{P}{Q} \frac{\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45}{\pm 1, \pm 2}$$

$$\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45 \\ \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{9}{2}, \pm \frac{15}{2}, \pm \frac{45}{2}$$

VI. Write a polynomial function, given the zeros of the function.

1.  $x = [3, -3, 4]$

$$(x-3)(x+3)(x-4)$$

✓

$$(x^2-9)(x-4)$$

2.  $x = [6, -1, 2, 3]$

$$(x-6)(x+1)(x-2)(x-3)$$

✓

$$(x^2-5x-6)(x^2-5x+6)$$

$$f(x) = x^3 - 4x^2 - 9x + 36$$

$$x^4 - \underline{\underline{5x^3}} + \underline{\underline{6x^2}} - \underline{\underline{5x^3}} + \underline{\underline{25x^2}} - \underline{\underline{30x}}$$

$$\sim \sim \sim \sim \sim \sim$$
$$-6x^2 + 30x - 36$$

$$f(x) = x^4 - 10x^3 + 25x^2 - 36$$

VII. Given one zero of the polynomial function, find the other zeros.

$$1. \quad f(x) = x^3 + x^2 - 13x + 3; \quad x = 3$$

$$\begin{array}{r} 3 \\ \hline 1 & 1 & -13 & 3 \\ & 3 & 12 & -3 \\ \hline 1 & 4 & -1 & 0 \end{array}$$

$$\begin{aligned} a &= 1 \\ b &= +4 \\ c &= -1 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 + 4x - 1$$

$$= \frac{-4 \pm \sqrt{20}}{2}$$

$$x = \frac{-4 \pm \sqrt{20}}{2}$$

$$\rightarrow x = \frac{-4 \pm 2\sqrt{5}}{2}$$

$$\downarrow \quad x = -2 \pm \sqrt{5}$$

$$2. \quad g(x) = x^4 + 6x^3 - 4x^2 - 54x - 45; \quad x = -3 \text{ and } x = 3$$

$$\begin{array}{r} -3 \\ \hline 1 & 6 & -4 & -54 & -45 \\ & -3 & -9 & 39 & 45 \\ \hline 1 & 3 & -13 & -15 & 0 \end{array}$$

$$x^3 + 3x^2 - 13x - 15$$

$$\begin{array}{r} 3 \\ \hline 1 & 3 & -13 & -15 \\ & 3 & 18 & 15 \\ \hline 1 & 6 & 5 & 0 \end{array}$$

$$x^2 + bx + 5$$

$$(x+5)(x+1)$$

$$x = -5, x = -1$$

VIII. Find the real zeros of the polynomial function.

$$1. \quad f(x) = x^3 + x^2 - 5x + 3$$

$$\begin{array}{r} P = \pm 1, \pm 3 \\ Q = \pm 1 \end{array} \quad \pm 1, \pm 3$$

$$\begin{array}{r} x \\ \hline 1 & 1 & 0 & \checkmark \end{array}$$

$$\begin{array}{r} 1 & 1 & -5 & 3 \\ & 1 & 2 & -3 \\ \hline 1 & 2 & -3 & 0 \end{array}$$

$$\begin{aligned} x^2 + 2x - 3 \\ (x+3)(x-1) \end{aligned}$$

$$(x-1)(x+3)(x-1)$$

$$\begin{array}{r} x = 1 \\ x = -3 \\ x = 1 \end{array}$$

$$2. \quad g(x) = x^3 - 13x^2 + 23x - 11$$

$$\begin{array}{r} P = \pm 1, \pm 11 \\ Q = \pm 1 \end{array} \quad \pm 1, \pm 11$$

$$\begin{array}{r} x \\ \hline 1 & 1 & 0 & \checkmark \end{array}$$

$$\begin{array}{r} 1 & -13 & 23 & -11 \\ & 1 & -12 & 11 \\ \hline 1 & -12 & 11 & 0 \end{array}$$

$$\begin{aligned} (x-1)(x^2 - 12x + 11) \\ (x-1)(x-11)(x-1) \end{aligned}$$

$$\begin{array}{r} x = 1 \\ x = 11 \\ x = 1 \end{array}$$

3.  $h(x) = x^3 + 4x^2 + 5x + 2$

$$\frac{P \pm 1, \pm 2}{Q \pm 1}$$

$$\begin{array}{r|rrrr} x & 1 & 4 & 5 & 2 \\ \hline -1 & & -1 & -3 & -2 \\ & 1 & 3 & 2 & 0 \end{array}$$

$$(x+1)(x^2+3x+2)$$

$$(x+1)(x+2)(x+1)$$

$$\boxed{x = -1 \\ x = -2 \\ x = -1}$$

4.  $h(x) = 4x^3 - 9x^2 + 6x - 1$

$$\frac{P \pm 1}{Q \pm 1, \pm 2, \pm 4}$$

$$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}$$

$$\begin{array}{r|rrrr} x & 4 & -9 & 6 & -1 \\ \hline 1 & & 4 & -5 & 1 \\ & 4 & -5 & 1 & 0 \end{array}$$

$$(x-1)(4x^2-5x+1)$$

$$\frac{P(4)|S(-5)}{-4, -1} \checkmark$$

$$(4x^2-4x)(-x+1) \\ 4x(x-1)-1(x-1) \\ (x-1)(4x-1)(x-1)$$

$$\boxed{x = 1 \\ x = \frac{1}{4} \\ x = 1}$$

6.  $h(x) = 5x^3 + 29x^2 + 19x - 5$

$$\pm 1, \pm \frac{1}{5}, \pm 5$$

$$\begin{array}{r|rrrr} -1 & 5 & 29 & 19 & -5 \\ & & -5 & -24 & 5 \\ & 5 & 24 & -5 & 0 \end{array}$$

$$(x+1)(5x^2+24x-5)$$

$$\frac{P(-25)|S(24)}{-1, 25} \checkmark$$

$$(5x^2+25x)(-x-5)$$

$$5x(x+5)-1(x+5)$$

$$(x+1)(5x-1)(x+5)$$

$$\boxed{x = -1, x = \frac{1}{5}, x = -5}$$

7.  $h(x) = 3x^3 + 11x^2 + 5x - 3$

$$\frac{P \pm 1 \pm 3}{Q \pm 1 \pm 3}$$

$$\begin{array}{r|rrr} x & 3 & 11 & 5 \\ \hline -1 & & -3 & -8 \\ & 3 & 8 & -3 \end{array}$$

$$\begin{array}{r|rrr} -1 & 3 & 11 & 5 \\ & & -3 & -8 \\ & 3 & 8 & -3 \end{array}$$

$$(x+1)(3x^2+8x-3)$$

$$\frac{P(-9)|S(8)}{-1, 9} \checkmark$$

$$(3x^2+9x)(-x-3)$$

$$3x(x+3)-1(x+3) \\ (x+1)(3x-1)(x+3)$$

5.  $h(x) = 5x^4 - 46x^3 + 84x^2 - 50x + 7$

$$\frac{P \pm 1, \pm 7}{Q \pm 1, \pm 5}$$

$$\pm 1, \pm \frac{1}{5}, \pm 7, \pm \frac{7}{5}$$

$$\begin{array}{r|rrrr} x & 1 & 0 & 0 & \checkmark \\ \hline 1 & 0 & 0 & 0 & \checkmark \\ 7 & 0 & 0 & 0 & \checkmark \end{array}$$

$$\begin{array}{r|rrrr} 1 & 5 & -46 & 84 & -50 & 7 \\ & & 5 & -41 & 43 & -7 \\ & 5 & -41 & 43 & -7 & 0 \end{array}$$

$$5x^3-41x^2+43x-7$$

$$\begin{array}{r|rrr} 7 & 5 & -41 & 43 & -7 \\ & & 35 & -42 & 7 \\ & 5 & -6 & 1 & 0 \end{array}$$

$$(x-1)(x-7)(5x^2-6x+1)$$

$$(x-1)(x-7)(5x-1)(x-1)$$

$$\boxed{x = 1, x = 7, x = \frac{1}{5}, x = 1}$$

8.  $h(x) = 3x^4 - 10x^3 - 24x^2 - 6x + 5$

$$\frac{P \pm 1, \pm 5}{Q \pm 1, \pm 3}$$

$$\begin{array}{r|rrrr} x & 3 & -10 & -24 & -6 & 5 \\ \hline -1 & 3 & 13 & 11 & -5 \\ & 3 & -13 & -11 & 5 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 3 & -10 & -24 & -6 & 5 \\ & & -3 & 13 & 11 & -5 \\ & 3 & -13 & -11 & 5 & 0 \end{array}$$

$$3x^3-13x^2-11x+5$$

$$\begin{array}{r|rrr} 5 & 3 & -13 & -11 & 5 \\ & & 15 & 10 & -5 \\ & 3 & 2 & -1 & 0 \end{array}$$

factor

$$\begin{array}{r|rrr} & 3 & -13 & -11 & 5 \\ & & 15 & 10 & -5 \\ & 3 & 2 & -1 & 0 \end{array}$$

$$(x+1)(x-5)(3x^2+2x-1) \\ (x+1)(x-5)(3x-1)(x+1)$$

$$\boxed{x = -1, x = 5, x = \frac{1}{3}, x = -1}$$