

Key

Solving Trigonometric Equations:

Undoing:

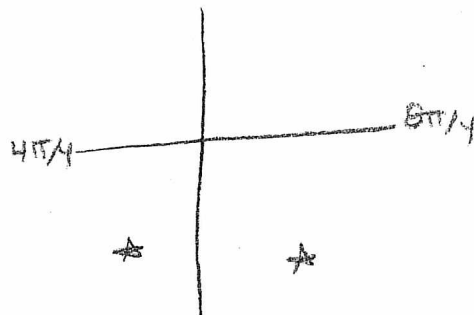
$$\sqrt{2} \sin x + 1 = 0$$

$$\frac{\sqrt{2} \sin x}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$$x = 5\pi/4, 7\pi/4$$

$$\sin x = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\sin x = -\sqrt{2}/2 \quad (\pi/4)$$



Factoring (backwards FOIL):

$$\sec^2 x - \sec x - 2 = 0$$

$$\cos x = 1/2 \quad (\pi/3)$$



$$x = \pi/3, 5\pi/3$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$(\sec x - 2)(\sec x + 1) = 0$$

$$\sec x - 2 = 0$$

$$\sec x = 2$$

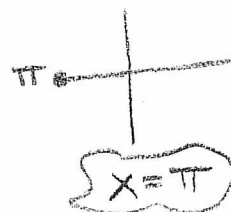
$$\cos x = 1/2$$

$$\sec x + 1 = 0$$

$$\sec x = -1$$

$$\cos x = -1$$

$$\cos x = -1$$



$$x = \pi$$

Factoring GCF:

$$3 \tan^3 x = \tan x$$

$$3 \tan^3 x - \tan x = 0$$

$$\tan x (3 \tan^2 x - 1) = 0$$

$$\tan x = 0$$



$$x = 0, \pi$$

$$3 \tan^2 x - 1 = 0$$

$$3 \tan^2 x = 1$$

$$\tan^2 x = 1/3$$

$$\tan x = \pm \frac{\sqrt{1}}{\sqrt{3}}$$

$$\tan x = \pm \sqrt{3}/3 \quad (\pi/6)$$



$$x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$$

Using Identities: $2 \sin^2 x = 2 + \cos x$

$$2(1 - \cos^2 x) = 2 + \cos x$$

$$2 - 2 \cos^2 x = 2 + \cos x$$

$$-2 \cos^2 x = \cos x$$

$$+ 2 \cos^2 x + 2 \cos^2 x$$

$$2 \cos^2 x + \cos x$$

$$\cos x (2 \cos x + 1) = 0$$

$$\cos x = 0$$

$$x = \pi/2, 3\pi/2$$

$$2 \cos x + 1 = 0$$

$$2 \cos x = -1$$

$$\cos x = -1/2 \quad (\pi/3)$$



$$x = 2\pi/3, 4\pi/3$$

$$\tan x = \pm 1/\sqrt{3} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3} = \tan x \quad (\pi/6)$$

Square both sides ...CHECK for EXTRANEIOUS SOLUTIONS!

check!

$$\sec 0 = 1 - \tan 0$$

$$1 = 1 - 0$$

$$1 = 1 \quad \checkmark$$

$$\sec \pi = 1 - \tan \pi$$

$$-1 = 1 - 0$$

$$-1 \neq 1 \quad \text{Not}$$

Multiple Angles:

$$\tan^2 3x = 3$$

$$\sqrt{\tan^2 3x} = \sqrt{3}$$

$$\tan 3x = \pm \sqrt{3} \quad (\pi/3)$$

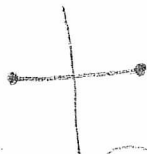
$$3x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{17\pi}{3}$$

1 rev 2 rev 3 rev

multiply
by 1/3

$$x = \pi/9, 2\pi/9, 4\pi/9, 5\pi/9, 7\pi/9, 8\pi/9, 10\pi/9, 11\pi/9, 13\pi/9, 14\pi/9, 16\pi/9, 17\pi/9$$

$$\tan x = 0$$



$$x = 0, \pi$$

$$(\sec x)^2 = (1 - \tan x)^2$$

$$\sec^2 x = (1 - \tan x)(1 + \tan x)$$

$$\sec^2 x = 1 - \tan x - \tan x + \tan^2 x$$

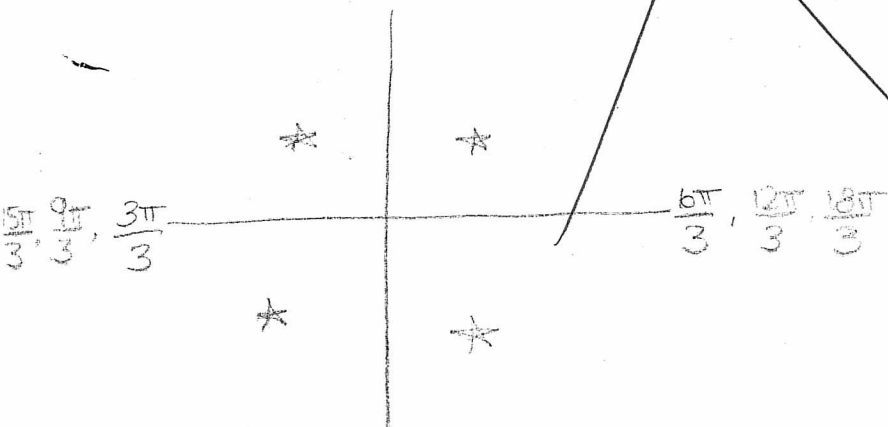
$$\sec^2 x = 1 - 2\tan x + \tan^2 x$$

↓

$$1 + \tan^2 x = 1 - 2\tan x + \tan^2 x$$

$$0 = -2\tan x$$

$$\tan x = 0$$



Additional Problems

1.) $2\cos^2 x - \cos x = 1$

$$2\cos^2 x - \cos x - 1 = 0$$

$$2x^2 - x - 1 = 0$$

$$P(-2) | S(-1)$$

$$-2, 1 \quad \checkmark$$

$$(2x^2 - 2x)(x - 1) = 0$$

$$2x(x - 1) + 1(x - 1) = 0$$

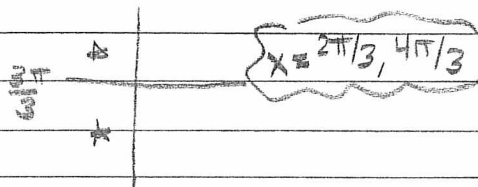
$$(2x + 1)(x - 1) = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$$2\cos x + 1 = 0$$

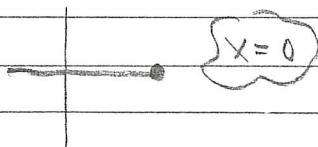
$$2\cos x = -1$$

$$\cos x = -1/2 \quad (\pi/3)$$



$$\cos x - 1 = 0$$

$$\cos x = 1$$



2.) $2\sin^2 x - 3\sin x = -1$

$$2\sin^2 x - 3\sin x + 1 = 0$$

$$2x^2 - 3x + 1 = 0$$

$$P(2) | S(-3)$$

$$-2, -1 \quad \checkmark$$

$$(2x^2 - 2x)(-x + 1) = 0$$

$$2x(x - 1) - 1(x - 1) = 0$$

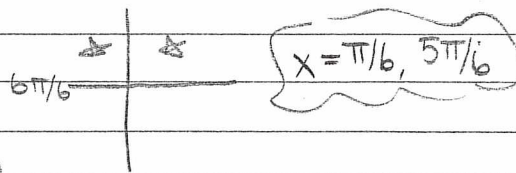
$$(2x - 1)(x - 1) = 0$$

$$(2\sin x - 1)(\sin x - 1) = 0$$

$$2\sin x - 1 = 0$$

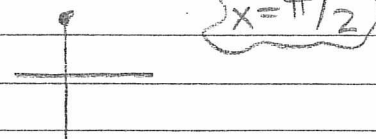
$$2\sin x = 1$$

$$\sin x = 1/2 \quad (\pi/6)$$



$$\sin x - 1 = 0$$

$$\sin x = 1$$



3.) $\cos^2 x + \sin x = 1$

↓

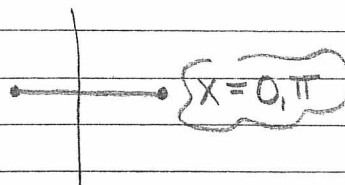
$$1 - \sin^2 x + \sin x = 1$$

$$\frac{-\sin^2 x + \sin x}{-1 \quad -1} = 0$$

$$\sin^2 x - \sin x = 0$$

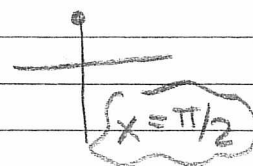
$$\sin x (\sin x - 1) = 0$$

$$\sin x = 0$$



$$\sin x - 1 = 0$$

$$\sin x = 1$$



$$4.) \sin^2 x + 2\cos x = 2$$

$$\begin{array}{r} \downarrow \\ 1 - \cos^2 x + 2\cos x = 2 \\ -2 \qquad \qquad \qquad -2 \\ \hline -\cos^2 x + 2\cos x - 1 = 0 \\ -1 \qquad \qquad \qquad -1 \end{array}$$

$$\cos^2 x - 2\cos x + 1 = 0$$

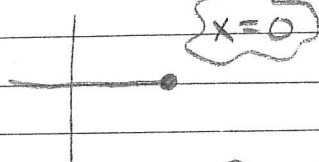
$$x^2 - 2x + 1 = 0$$

$$(x-1)(x-1) = 0$$

$$(\cos x - 1)(\cos x - 1) = 0$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$

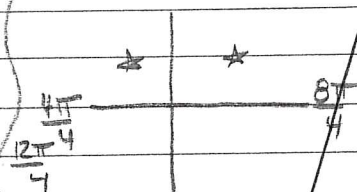


$$5.) 2\sin 2x - \sqrt{2} = 0$$

$$2\sin 2x = \sqrt{2}$$

$$\sin 2x = \sqrt{2}/2 \quad (\pi/4)$$

$$2x = \frac{\pi/4, 3\pi/4}{1 \text{ rev}}, \quad \frac{9\pi/4, 11\pi/4}{2 \text{ rev}}$$



$$x = \pi/8, 3\pi/8, 9\pi/8, 11\pi/8$$

$$\frac{\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{8}$$

$$\frac{9\pi}{4} \cdot \frac{1}{2} = \frac{9\pi}{8}$$

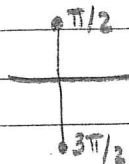
$$\frac{3\pi}{4} \cdot \frac{1}{2} = \frac{3\pi}{8}$$

$$\frac{11\pi}{4} \cdot \frac{1}{2} = \frac{11\pi}{8}$$

$$6.) \cos 4x (\cos x - 1) = 0$$

$$\cos 4x = 0$$

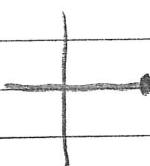
$$4x = \frac{\pi/2, 3\pi/2}{1 \text{ rev}}, \quad \frac{5\pi/2, 7\pi/2}{2 \text{ rev}}, \quad \frac{9\pi/2, 11\pi/2}{3 \text{ rev}}, \quad \frac{13\pi/2, 15\pi/2}{4 \text{ rev}}$$



$$x = \pi/8, 3\pi/8, 5\pi/8, 7\pi/8, 9\pi/8, 11\pi/8, 13\pi/8, 15\pi/8$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$



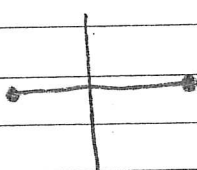
$$54.) \sqrt{3} \tan 3x = 0$$

$$\frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan 3x = 0$$

$$3x = \frac{0, \pi}{1 \text{ rev}}, \quad \frac{2\pi, 3\pi}{2 \text{ rev}}, \quad \frac{4\pi, 5\pi}{3 \text{ rev}}$$

$$x = 0, \pi/3, 2\pi/3, \pi, 4\pi/3, 5\pi/3$$



$$7.) 3\csc^2 5x = -4$$

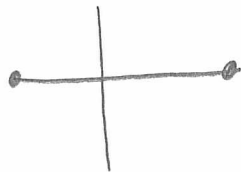
$$\sqrt{\csc^2 5x} = \sqrt{-4/3}$$

NO SOLUTION

$$8.) \sin^2 x - 2\sin x = 0$$

$$\sin x (\sin x - 2) = 0$$

$$\sin x = 0$$



$$x = 0, \pi$$

$$\sin x - 2 = 0$$

$$\sin x \neq 2$$

$$9.) 2\cos^2 x + 3\cos x = 0$$

$$\cos x (2\cos x + 3) = 0$$

$$\cos x = 0$$



$$x = \pi/2, 3\pi/2$$

$$2\cos x + 3 = 0$$

$$2\cos x = -3$$

$$\cos x \neq -3/2$$