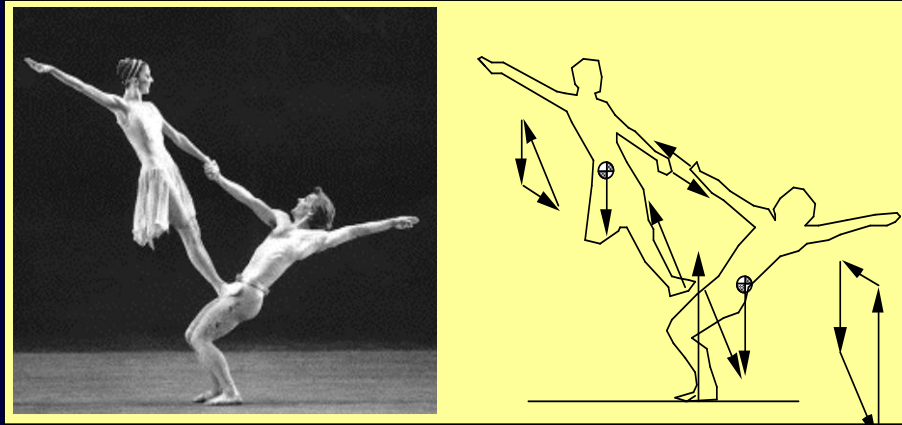# The Physics of Dance

# Today...

- Introduction

- Some physics vocabulary

- Net force: adding forces

- More physics vocabulary

- Static balance

- Balance while turning

- The physics of turns

- Jumps (and black holes...)

# Introduction

How'd we get ourselves into this?

- George's story
- Dede's story

Some statements about dance technique can be restated using a physics vocabulary…

Lifts "feel" the way they do for the same reason that a large star will collapse to form a black hole...

# Some Physics Vocabulary

- Velocity ($\vec{v}$): *how fast and in what direction* something is moving.

- Momentum($\vec{p}$): The magnitude of an object's momentum gives an indication of how hard it is to bring to a stop.

$$\text{momentum} = \text{mass} \times \text{velocity}: \vec{p} = m\vec{v}$$

- Force: Just like you'd think. It's a push, characterized by a magnitude (how hard) and a direction.
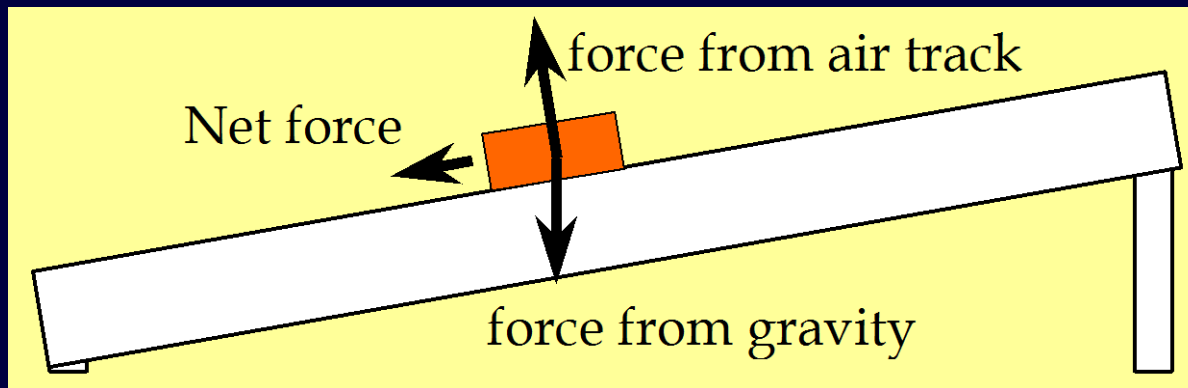
# Force changes momentum

- The harder you push the more rapidly the momentum changes.

- The longer you push the greater the total change in momentum.

- The equation: $\Delta\vec{p} = \vec{F}\,\Delta t$

A force can change an object's speed and/or direction of motion.

# Adding forces

Cart feels two forces:
- gravity: downwards
- push from track: up & to-the-left



The *net force* causes the change in the cart's momentum.

Forces on a dancer: gravity (down), support from floor (up), friction from floor (sideways)

# More Physics Vocabulary

- Angular velocity ($\vec{\omega}$): how fast something spins *and in what direction* its rotation axis points.

- Angular momentum($\vec{L}$): The magnitude of an object's angular momentum gives an indication of how hard it is to bring to a stop.
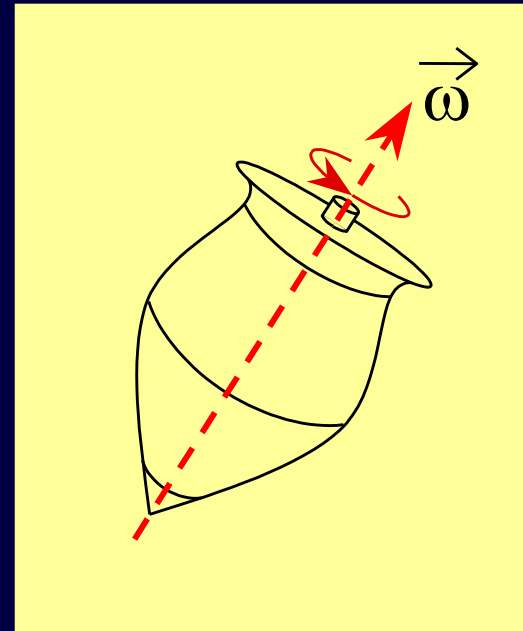
# Even More Physics
## Vocabulary

- Large rotational inertia, spinning slowly or small rotational inertia, spinning rapidly… both are hard to stop.
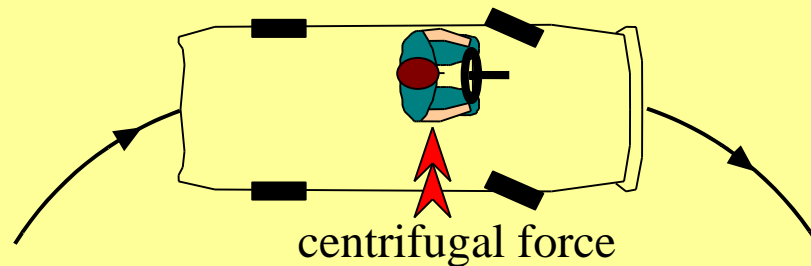
angular momentum = rotational inertia × angular velocity

$$\vec{L} = \vec{I}\,\vec{\omega}$$

- Angular velocity
- angular momentum
- rotational inertia



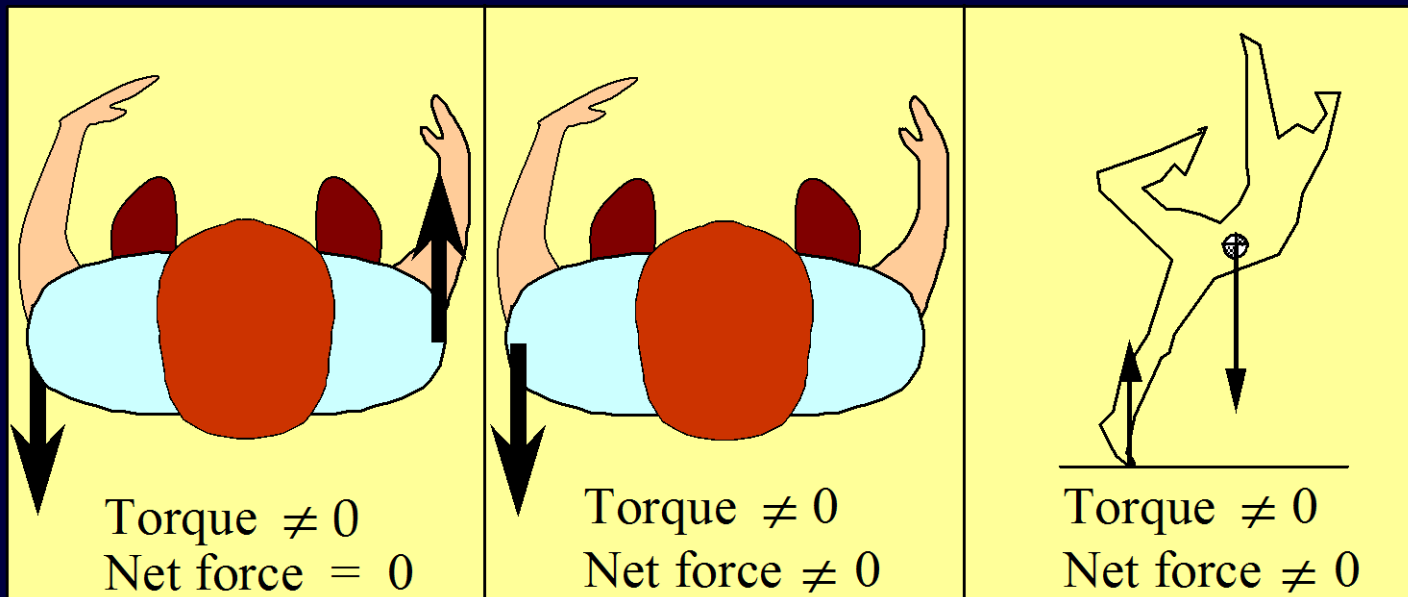$\vec{\omega}$

# Still More Physics Vocabulary

- Centrifugal force:  Effective force pushing something to the outside of a turn.



centrifugal force

doubling ω quadruples the centrifugal force…
doubling the turn's radius doubles the force.

# Even Still More Physics Vocabulary

- Torque ($\vec{\tau}$): "off center" forces which can cause something to spin.



Torque $\neq 0$
Net force $= 0$

Torque $\neq 0$
Net force $\neq 0$

Torque $\neq 0$
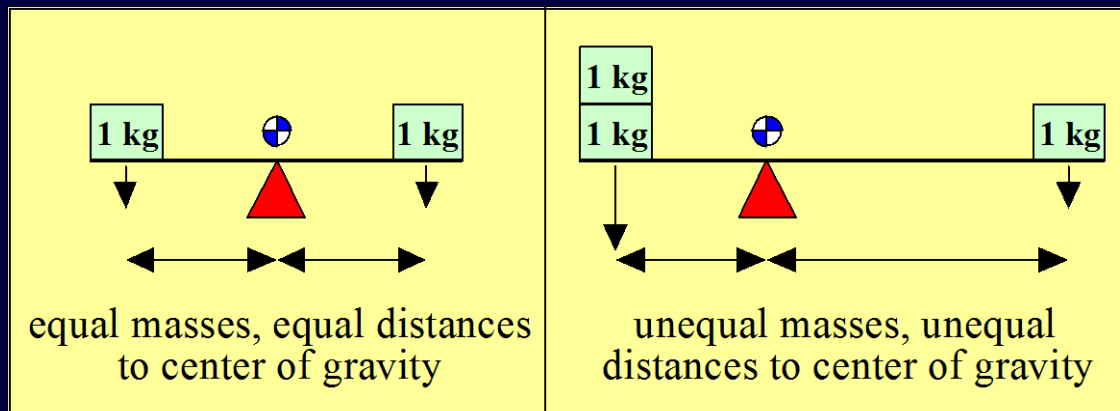Net force $\neq 0$

# About torque...

Torque changes angular momentum:

- The harder you push, the more quickly the object "spins up."
- The further from the spin axis you apply the forces, the more quickly the object spins up.
- The longer you push, the greater the total change in angular momentum.
- If no torque is applied to a rigid object, its spin axis doesn't wobble.

11

# Static Balance

We all know what it means...  in physics terms:

- Net force is zero (otherwise your momentum would change: you might fall)
- Net torque is zero (otherwise your angular momentum would change: you might tip over)



equal masses, equal distances to center of gravity

unequal masses, unequal distances to center of gravity

# Center of gravity

Just like you'd think.  You can balance around this point.

Reason for the name: net effect of gravity on an object (a dancer) is the same as if it were only acting at the c.g., and not on all parts of the object.
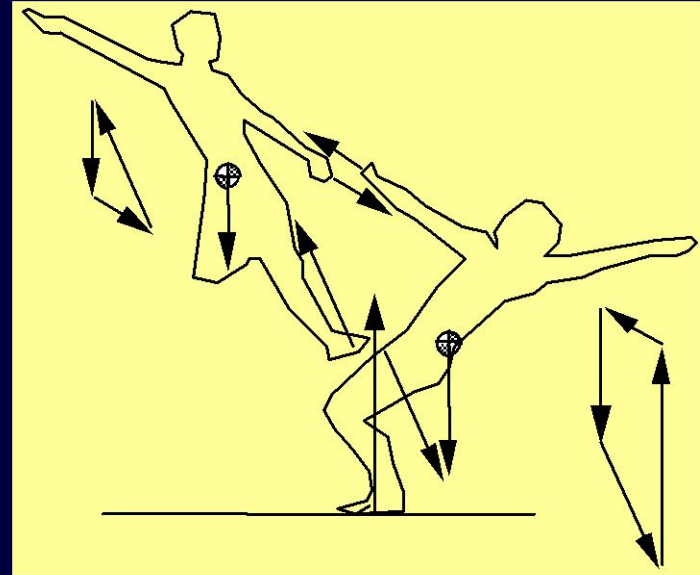
# Forces on a Balanced Dancer

Gravity (pulls down, acts at c.g.) + support from floor (pushes up, acts at feet)
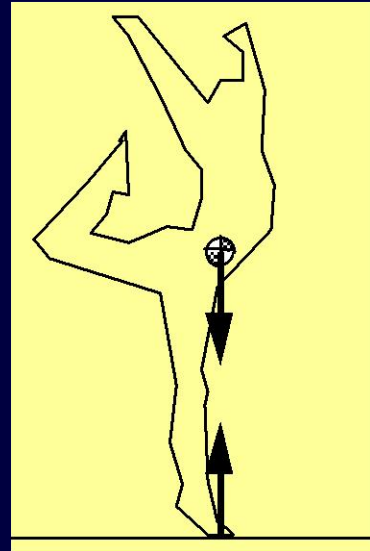


Keith Roberts (ABT) in *The Rite of Spring,* photo by Roy Round (New York)

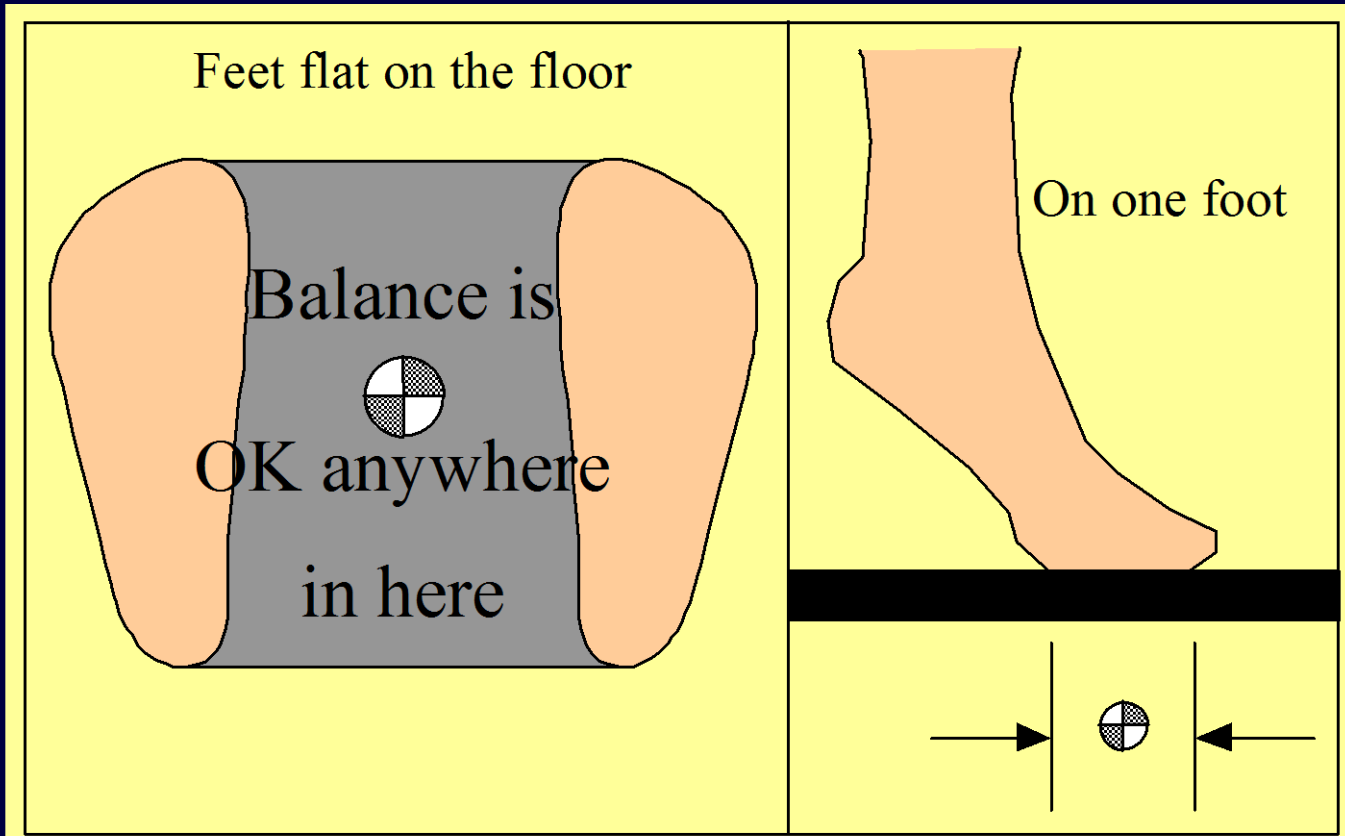Forces sum to zero; both forces "point through" the dancer's center of gravity, so torque is zero.

# More complicated example



Forces on each dancer sum to zero. Though some forces are "off-center" torques also sum to zero.

# Staying Balanced



Feet flat on the floor

Balance is OK anywhere in here

On one foot

Staying balanced: center of gravity remains directly above the area of contact with the floor.  (Otherwise  $\vec{\tau} \neq 0$:  you'd tip over.)

# Staying Balanced

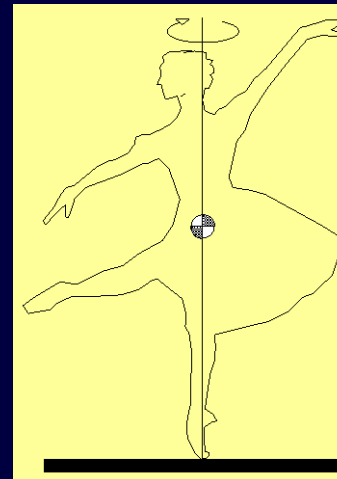Smaller floor contact area (*en pointe*, for example) makes balance more difficult.
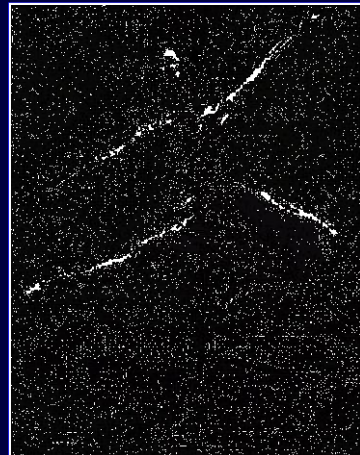
Maintaining balance:

- shift foot to move floor contact area
- adjust arms/legs/torso to move c.g.

# Balance While Turning

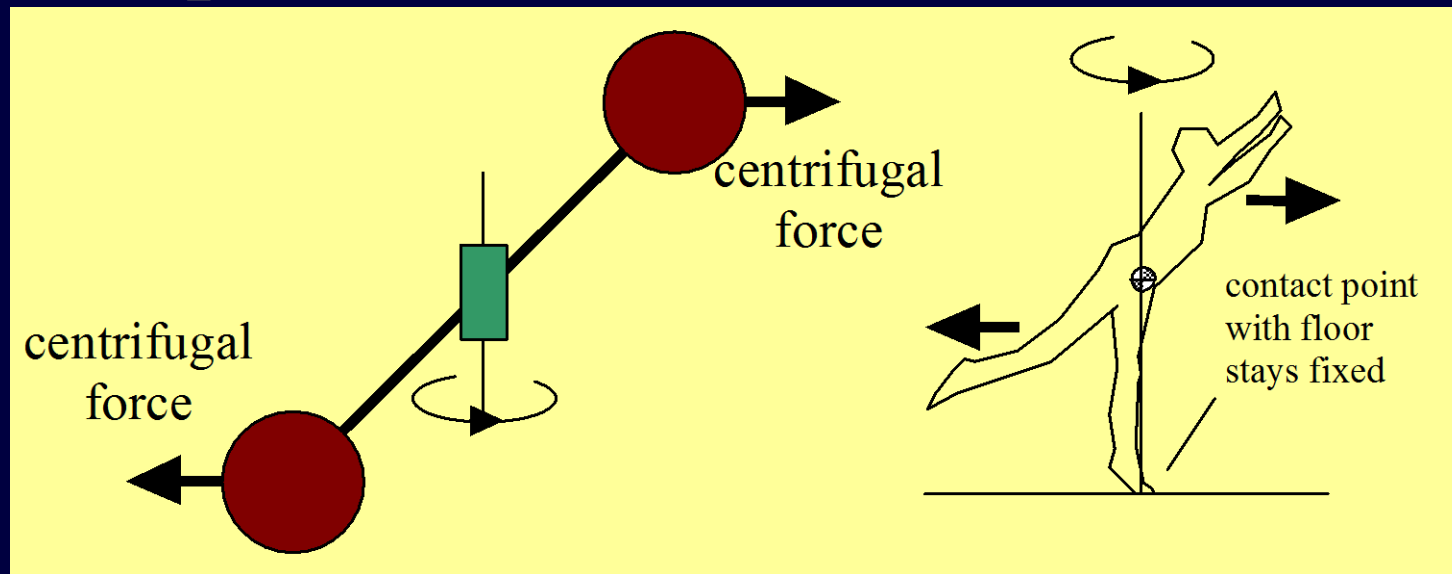Two distinct "physics domains" here:

- slow turns: maintain static balance

- fast turns: rotation axis shouldn't wobble (much)

# Balance While Turning

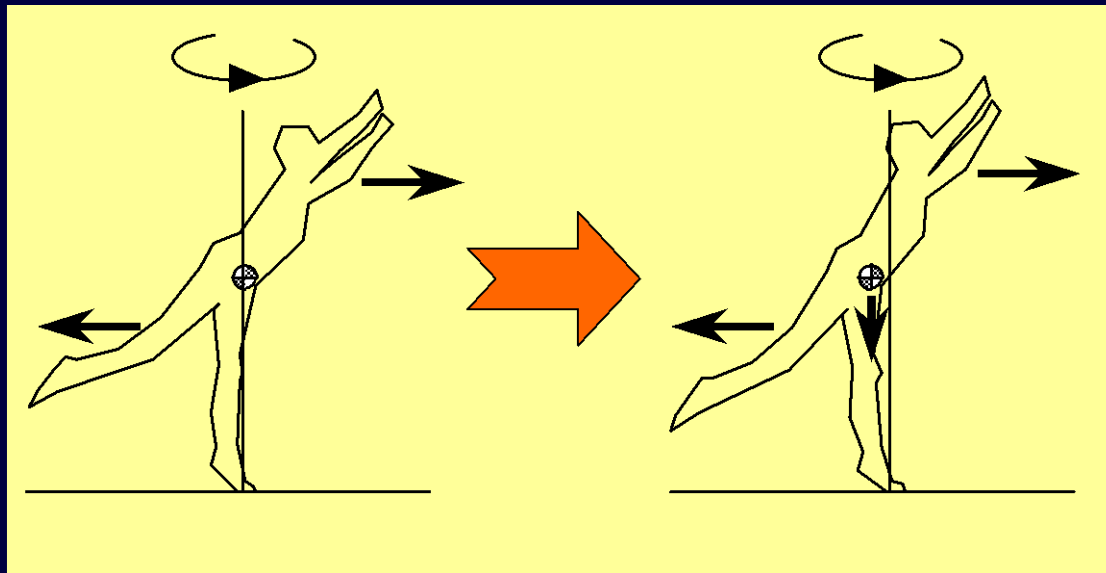Not all positions which are statically balanced can be held in a turn.

An example:



Centrifugal forces exert a torque, throwing the dancer off balance.

# Balance While Turning

Shift center of gravity to maintain dynamic (but not static) balance:

# The Physics of Turns

Reminders:

- $\vec{L}$  is angular momentum
- $\overleftrightarrow{I}$  is rotational inertia
- $\vec{\omega}$  is angular velocity
- $\vec{\tau}$  is torque
- $\vec{L} = \overleftrightarrow{I}\,\vec{\omega}$
- $\Delta\vec{L} = \vec{\tau}\Delta t$

# The Physics of Turns

$\vec{I}$ indicates how hard it is to start something spinning (just as mass is an indication of how hard it is to start something moving).

$\Delta\vec{L} = \vec{\tau}\Delta t$: if no torque is applied $\vec{L}$ must be constant.

$\vec{L} = \vec{I}\,\vec{\omega}\ldots$ if no torque is applied:
- Decreasing $\vec{I}$ will increase $\vec{\omega}$
- Increasing $\vec{I}$ will decrease $\vec{\omega}$

# Calculating Rotational Inertia $\vec{I}$

- Greater mass $\rightarrow$ greater $\vec{I}$. (Doubling the mass doubles $\vec{I}$.)

- Mass placed farther from rotation axis $\rightarrow$ greater $\vec{I}$. (Doubling the distance quadruples $\vec{I}$.)

- $\vec{I} = mr^2$ (one mass)

- $\vec{I} = m_1 r_1^2 + m_2 r_2^2 + \cdots$ (several masses)

# Increasing $\overleftrightarrow{I}$ Decreases Spin Rate



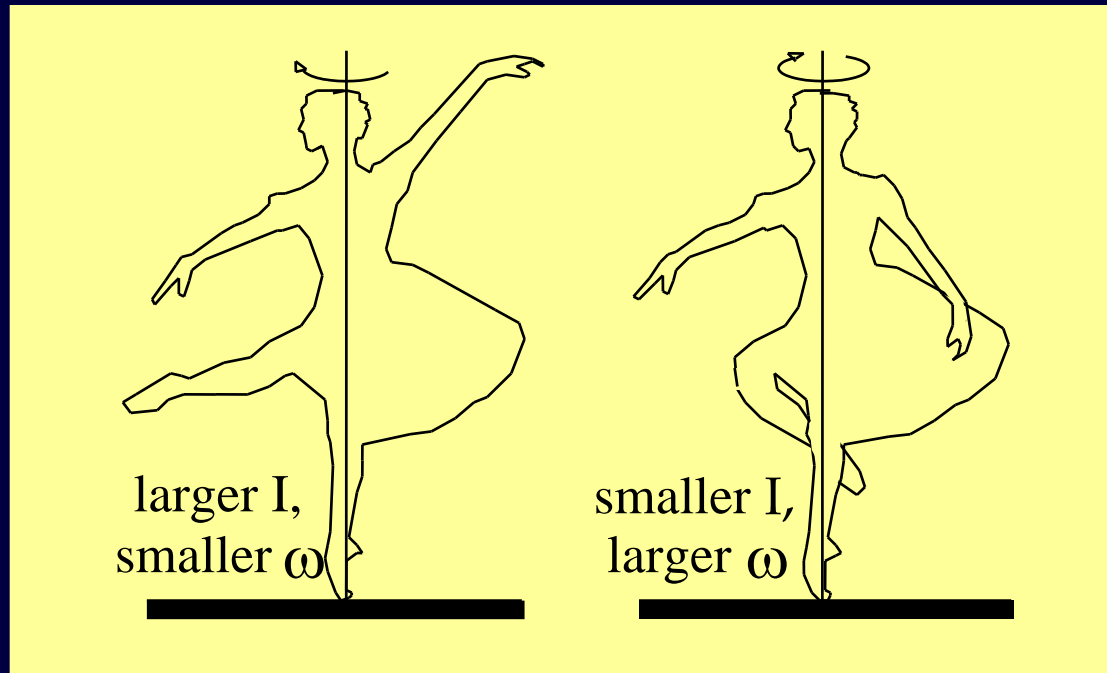| Before... | ...and after |
|---|---|
| r  4 rpm | r ×2  1 rpm |
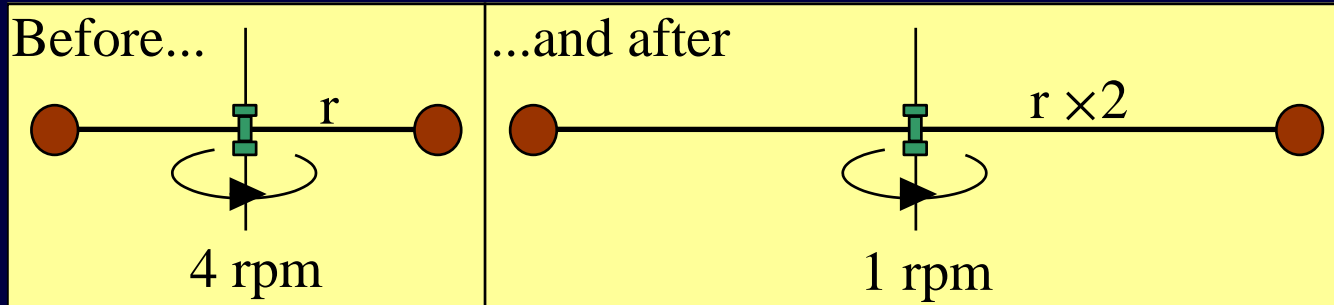
$\overleftrightarrow{I}$ quadruples in the figure.

Because of "conservation of angular momentum" you can change the speed of a turn when *en pointe* by extending or retracting arms and legs.
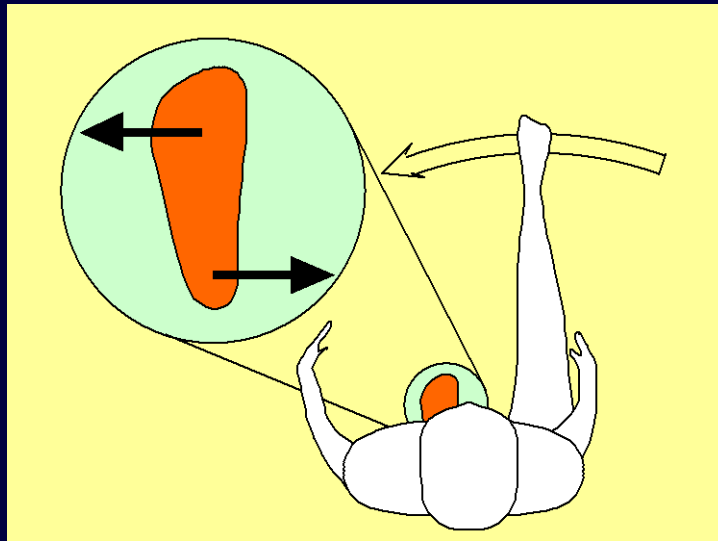
No torque is involved!

# Increasing $\vec{I}$ Decreases Spin Rate



larger I, smaller ω

smaller I, larger ω

# Friction and Torque

Friction between the floor and your foot (or feet) can also generate a torque. Swinging a leg going into a turn works like this:



You push on the floor one way, the floor pushes back the other way.

# The Physics of Jumps

Gravitational force depends on mass:

- More massive → greater gravitational pull

- Mass is to gravity like electric charge is to electrostatics. In electrostatics: the larger the electric charge something has, the greater the force it feels in an electric field.

Momentum also depends on mass: the greater your mass, the harder it is to stop moving at a particular speed. (Recall: $p = mv$)

# The Physics of Jumps

This is <u>very</u> curious.  Why should an object's momentum/velocity relationship have <u>anything at all</u> to do with the gravitational pull on that object?

(Doubling an object's mass both doubles its momentum and doubles the strength of the gravitational force it experiences.)

# The Physics of Jumps

The equivalence of "gravitational" and "inertial" mass gives rise to Einstein's General Theory of Relativity.  Black holes, curved spacetime, and so on!

If they were not equivalent, dance technique associated with jumps would need to be different from what it actually is.

# Gravity...

...is the only force acting on a dancer in mid-air.

Recall: net effect of gravity is the same as if it were only acting at the dancer's center of gravity.

Gravitational force: $\vec{F} = m\vec{g}$ ($\vec{g}$ is constant)

Force and momentum: $\Delta\vec{p} = m\Delta\vec{v} = \vec{F}\Delta t$
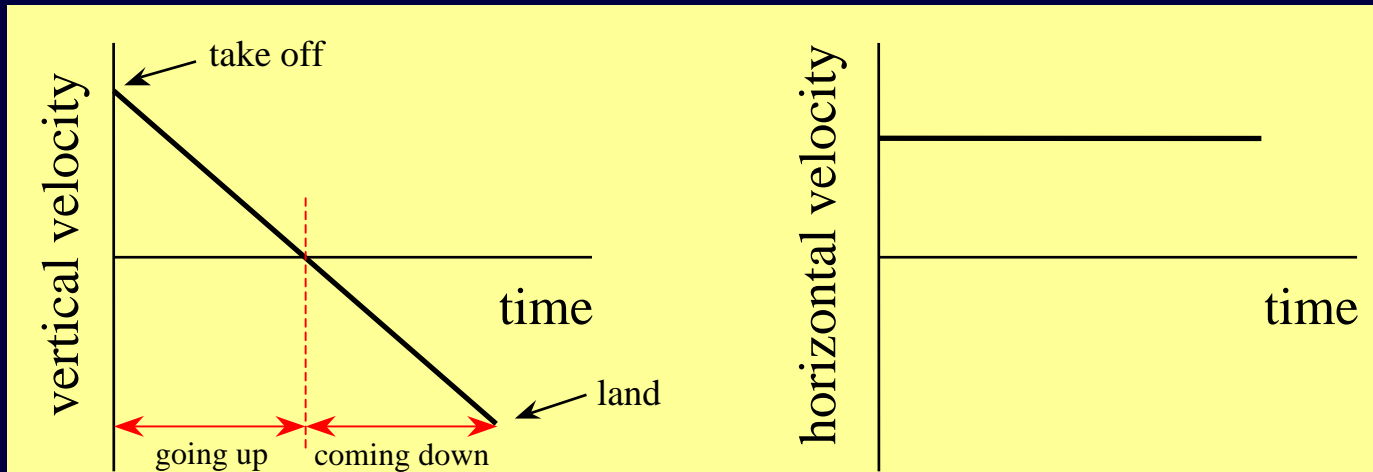
# Gravitational acceleration...

…is independent of mass.

Anything you drop speeds up at the same rate; anything thrown straight up slows down at the same rate.

Vertical speed changes by 32 feet per second (about 22 mph) for each second spent in the air.

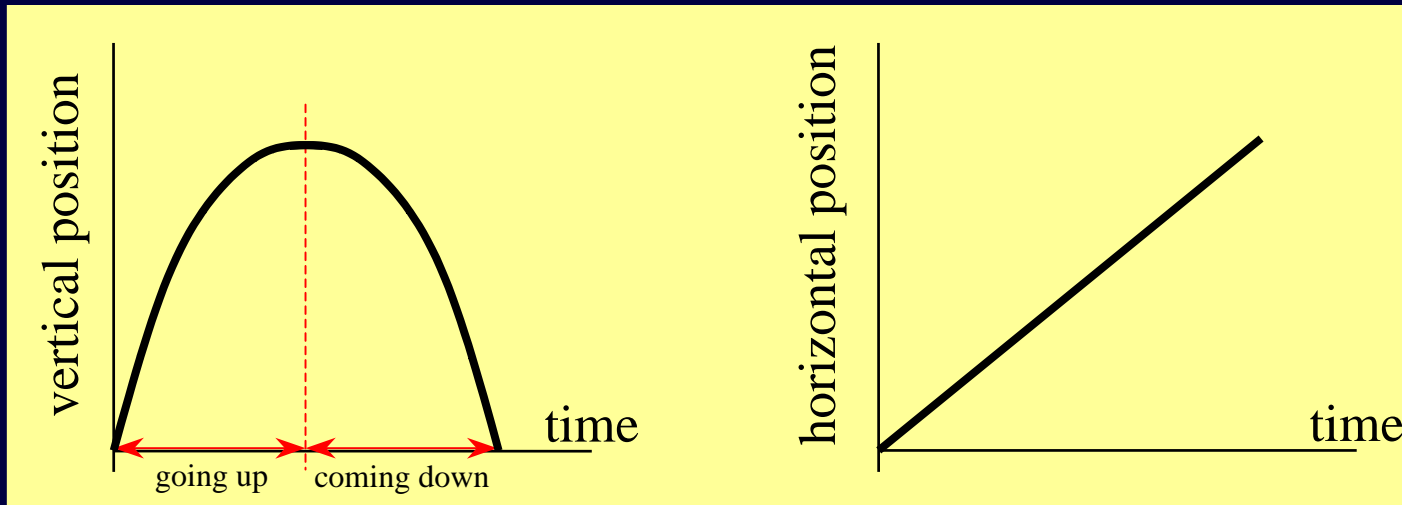# Gravity Only Influences Vertical (Not Horizontal) Motion

Velocity of a dancer's center of gravity *vs.* time during a jump:



Dancer's vertical motion slows, stops, changes direction as dancer rises, then falls.
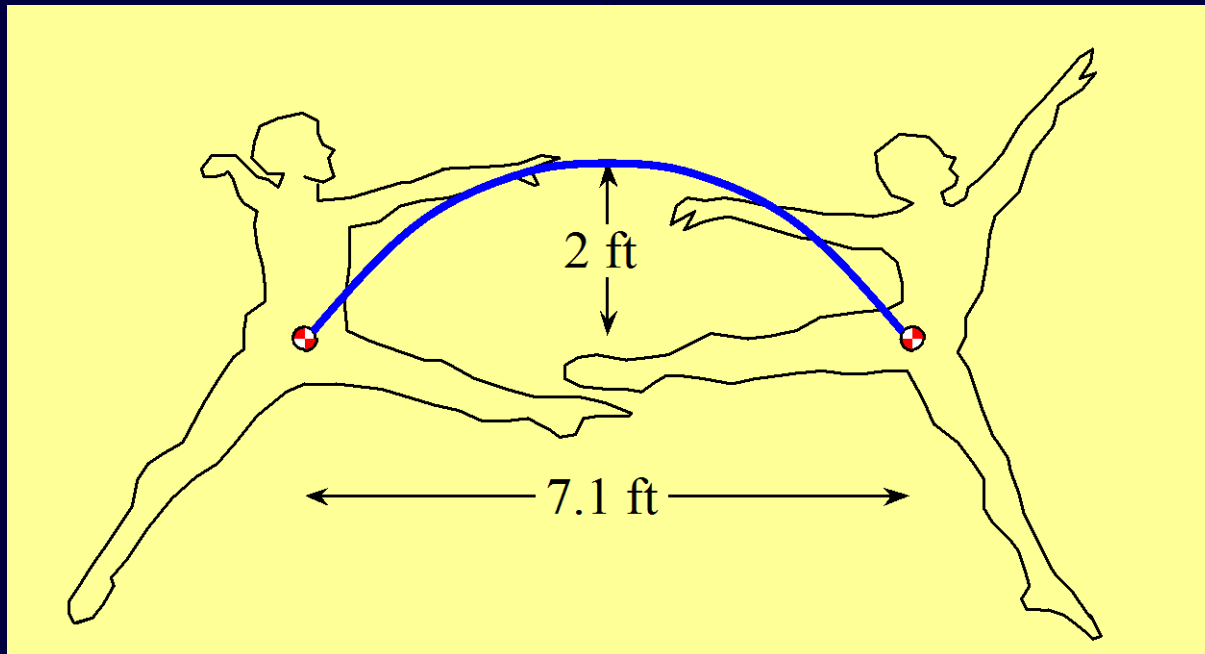
# Trajectory in a Jump

Height *vs.* time is a parabola; horizontal distance *vs.* time is a straight line:
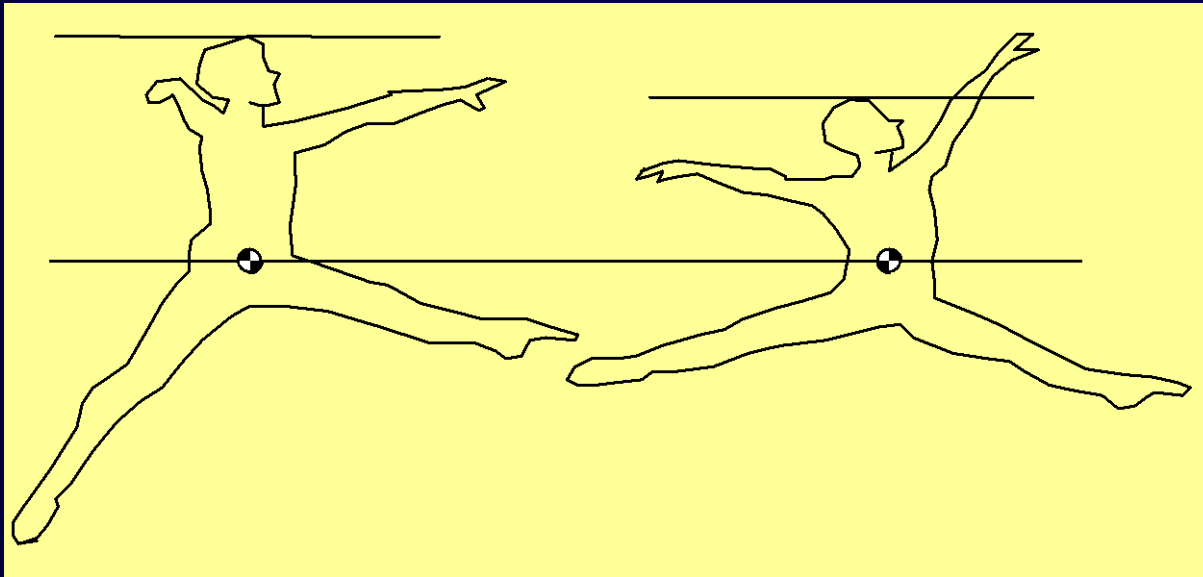
# Trajectory in a Jump

The dancer's center of gravity moves along a trajectory which is also a parabola. For a 2 foot rise in the height of the c.g. and a 10 ft/sec horizontal velocity we see the following:
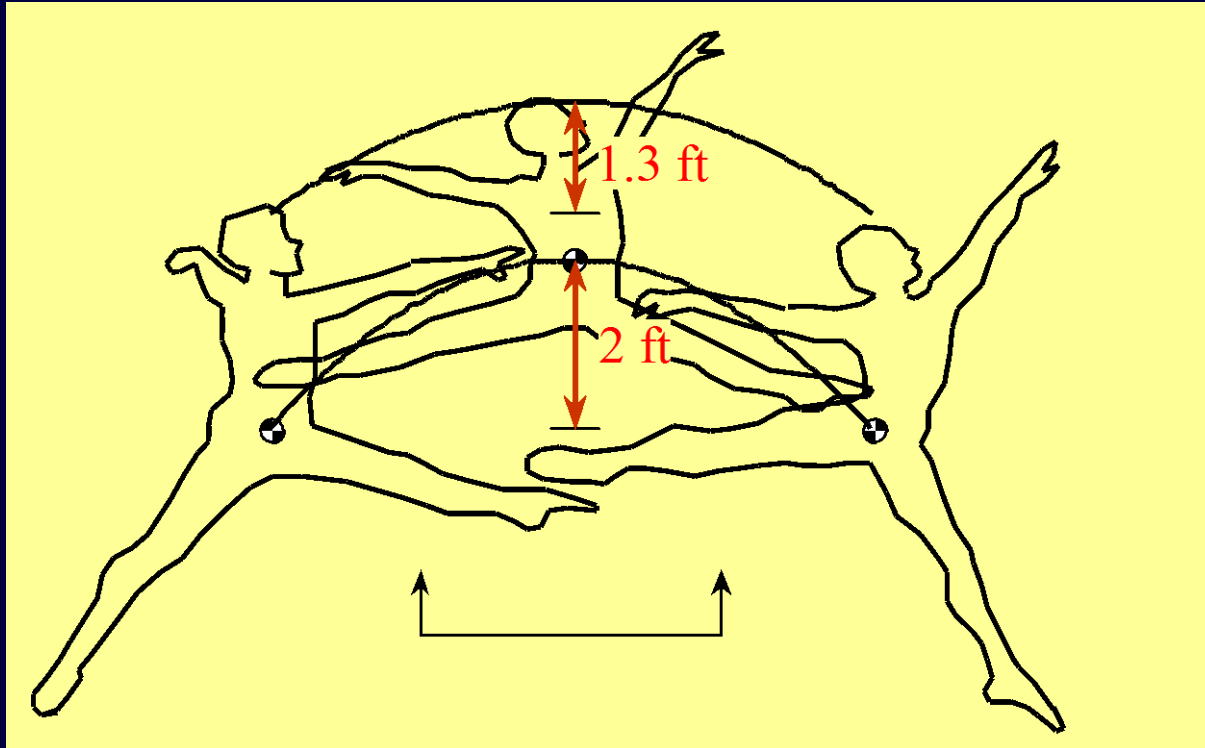


2 ft

7.1 ft

# Illusions in a Jump

By raising his/her legs (changing the distance between the c.g. and his/her head)...



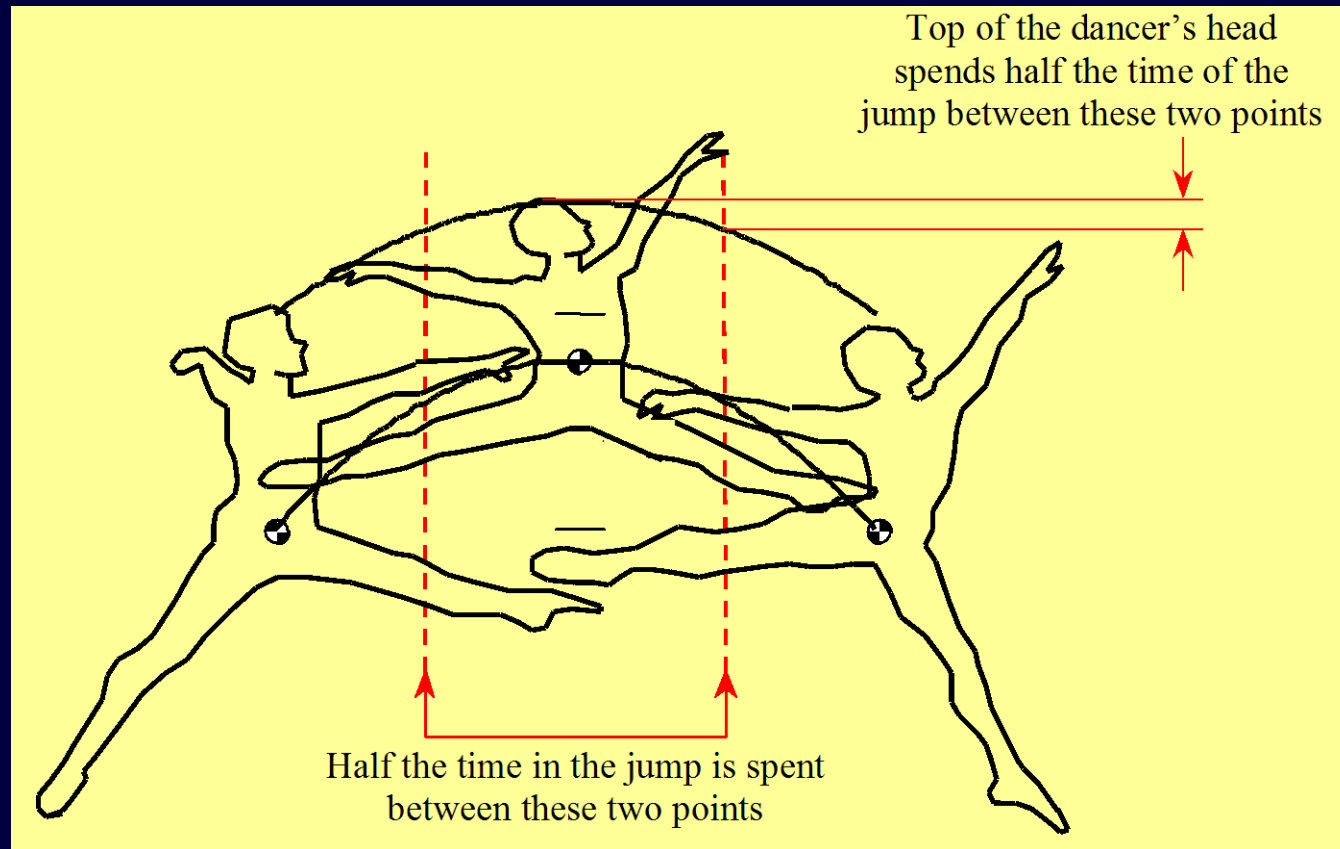The dancer can create an illusion of floating...

# Illusions in a Jump

The dancer can create an illusion of floating...



The vertical motion of the head is smaller than the vertical motion of the center of gravity. [36]

# Illusions in a Jump



Top of the dancer's head spends half the time of the jump between these two points

Half the time in the jump is spent between these two points

Half the duration of the jump is spent between the points indicated by the arrows; the height of the dancer's center of gravity only changes by 6 inches during this time.

# Dance is an Art

- Physics does not "explain" dance

- The reaction of the audience to dance is not "enhanced" by knowing the physics

- There are interesting parallels between some ideas of current interest in the humanities and those of modern physics (*e.g.* the blurring of boundaries between a thing and its environment)

- It is interesting to see where dance/physics vocabularies overlap.

# With More Time...

Time permitting, it would be fun to investigate some of the following:

- dancing with a partner

- effects relating to body size

- impacts and stress injuries

- the connection between "good technique" and injury prevention

# Acknowledgements and References

Dance faculty at UIUC:

Linda Lehovec, Rebecca Nettl-Fiol, Sean Sullivan, Renée Wadleigh, Erica Wilson-Perkins

Dance faculty at Hope College (Michigan):

Linda Graham

More information:

Kenneth Laws and Cynthia Harvey, *Physics, Dance, and the Pas de Deux,* Schirmer Books, New York, 1994.

http://www.uiuc.edu/ph/www/g-gollin/dance/dance_physics.html