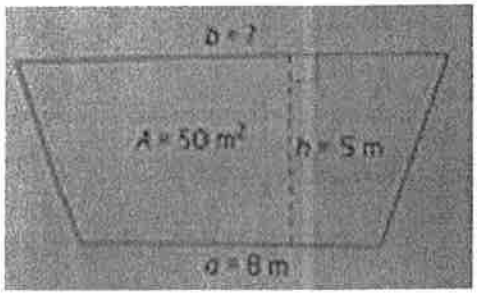
3.3 Solve Equations Involving Fractions

Minds-On

Kim is designing a rock garden in the shape of a trapezoid. She decides that the garden should have a front width of 8 m and a depth of 5 m. The area must be 50 m^2 to fit her design. How wide should Kim's garden be at the back?



Example 1:

Solve.

a) $6 = \frac{1}{3}(8 + x)$

b) $\frac{3(y-5)}{4} = 7$

Example 2:

Solve.

a) $\frac{k+2}{3} = \frac{k-4}{5}$

b) $\frac{1}{3}(2x - 5) = \frac{3}{4}(x - 2)$

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$A = \frac{(a+b)h}{2}$

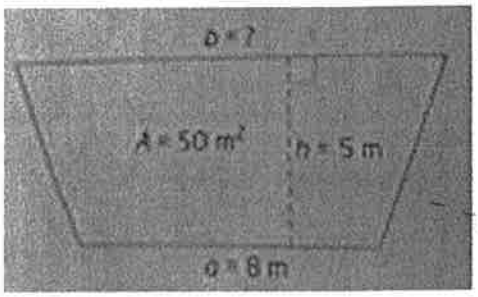
$2[50] = \left[\frac{(8+b)5}{2}\right]2$ $\quad +8,\ \times 5,\ \div 2$

$100 = (8+b)5$

$100 = 5(8+b)$ (A) $\rightarrow 100 = 40 + 5b$

$\frac{60}{5} = \frac{5b}{5}$

$b = 12$



back

front

∴ the back of the garden 12 m long.

Example 1:

Solve.

a) $\frac{1}{3}(6) = \left(\frac{1}{3}(8+x)\right)3$ $\quad +8,\ \div 3$

$18 = 8 + x$

$-8 \quad -8$

$10 = x$

$x = 10$

b) $4\left(\frac{3(y-5)}{4}\right) = (7)4$ $\quad -5+5,\ \times 3 \rightarrow \div 3,\ \div 4 \rightarrow \times 4$

$\frac{3(y-5)}{3} = \frac{28}{3}$

$y - 5 = 9.33\overline{3}$

$+5 \quad +5$

$y = 14.33\overline{3}$

Example 2:

Solve.

a) $\frac{k+2}{3} = \frac{k-4}{5}$

$\frac{15(k+2)}{3} = \frac{15(k-4)}{5}$

$5(k+2) = 3(k-4)$

$5k + 10 = 3k - 12$

$-3k \quad -3k$

$2k + 10 = -12$

$-10 \quad -10$

$2k = -22$

$k = -11$

b) $\frac{1}{3}(2x-5) = \frac{3}{4}(x-2)$

$\frac{12(2x-5)}{3} = \frac{12[3(x-2)]}{4}$

$4(2x-5) = 3(3(x-2))$

$8x - 20 = 3(3x-6)$

$8x - 20 = 9x - 18$

$+20 \quad +20$

$8x = 9x + 2$

$-9x \quad -9x$

4.3 Solve Equations Involving Fractions

Principles of Mathematics 9, pages 204–210

A

1. Solve using pencil and paper.

a) $\frac{1}{4}(x-3)=-2$

b) $6=-\frac{3}{5}(a-7)$

c) $\frac{m+7}{5}=3$

2. Solve using pencil and paper.

a) $9=\frac{3(k+4)}{2}$

b) $\frac{3k+5}{2}=10$

c) $1=\frac{2p-3}{5}$

3. Solve ~~using a Computer Algebra System (CAS). Use at least two steps~~.

a) $\frac{y-5}{3}=-4$

b) $\frac{1}{3}(p+2)=-5$

c) $3=\frac{4}{5}(h+2)$

4. Solve ~~using a CAS. Use at least two steps.~~

a) $5=\frac{4(n+3)}{2}$

b) $6=\frac{7-c}{2}$

c) $\frac{3+w}{-2}=4$

B

5. Solve ~~Find the root of each equation.~~ Check your answers.

a) $\frac{h-4}{5}=\frac{h-3}{6}$

b) $\frac{d-2}{4}=\frac{d+1}{3}$

c) $\frac{1}{3}(x+4)=\frac{1}{5}(x+2)$

6. Solve ~~Find the root of each equation.~~ Check your answers.

a) $\frac{1}{4}(p-7)=\frac{1}{6}(p-3)$

b) $\frac{2(k-5)}{3}=\frac{4(k+2)}{5}$

c) $\frac{3(s-4)}{4}=\frac{2(s-3)}{3}$

7. Solve ~~Find the root of each equation. Use a CAS to check your answers.~~

a) $\frac{2}{5}(3m+2)=\frac{3}{4}(m+5)$

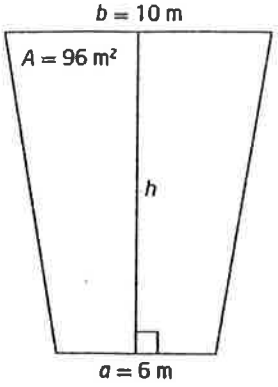
b) $\frac{2}{3}(k+2)=\frac{3}{4}(2k-1)$

c) $\frac{4c+5}{3}=\frac{2c+4}{5}$

d) $\frac{5-3n}{4}=\frac{2-n}{3}$

e) $\frac{2(3w+4)}{5}=\frac{2(2w-1)}{3}$

8. A trapezoidal deck has an area of 96 m^2. The front and back widths are 6 m and 10 m, as shown. What is the length of the deck from front to back?



9. Each solution contains an error. Identify the error and describe how to correct it.

a)
$$\frac{x+5}{4} = \frac{x-2}{3}$$
$$4(x+5) = 3(x-2)$$
$$4x + 20 = 3x - 6$$
$$4x + 20 - 3x - 20 = 3x - 6 - 3x - 20$$
$$x = -26$$

b)
$$\frac{1}{5}(2y+4) = \frac{1}{2}(y-3)$$
$$10 \times \frac{1}{5}(2y+4) = 10 \times \frac{1}{2}(y-3)$$
$$2y + 4 = y - 3$$
$$2y + 4 - y - 4 = y - 3 - y - 4$$
$$y = -7$$

10. Find the base of a triangle with height 8 cm and area 72 cm^2.

$A = \frac{b \times h}{2}$

C

11. The equation $F = \frac{9}{5}C + 32$ allows you to convert between Fahrenheit and Celsius temperatures. C is the temperature in degrees Celsius (°C) and F is the temperature in degrees Fahrenheit (°F).

a) The temperature at a resort is 30°C. What is this equivalent to in degrees Fahrenheit?

b) The temperature in the living room of a house is 77°F. What is this equivalent to in degrees Celsius?

12. Solve.

a) $\frac{2a}{3} + \frac{a-4}{5} = \frac{1}{2}$

b) $\frac{u+1}{2} + \frac{2u+3}{3} = \frac{u}{4}$

c) $\frac{w+3}{4} = \frac{w}{3} + \frac{2w-1}{5}$

Answers:

4.3 Solve Equations Involving Fractions, pages 73–74

1. **a)** $x = -5$ **b)** $a = -3$ **c)** $m = 8$

2. **a)** $k = 2$ **b)** $k = 5$ **c)** $p = 4$

3. **a)** $y = -7$ **b)** $p = -17$ **c)** $h = \frac{7}{4}$

4. **a)** $n = -\frac{1}{2}$ **b)** $c = -5$ **c)** $w = -11$

5. **a)** $h = 9$ **b)** $d = -10$ **c)** $x = -7$

6. **a)** $p = 15$ **b)** $k = -37$ **c)** $s = 12$

7. **a)** $m = \frac{59}{9}$ **b)** $k = \frac{5}{2}$ **c)** $c = -\frac{13}{14}$ **d)** $n = \frac{7}{5}$ **e)** $w = 17$

8. 12 m

9. **a)** The error is in the second line, $4(x + 5) = 3(x - 2)$. The numerators on each side of the first line were multiplied by their own denominators. The correct step should be to multiply both sides by 12 (the lowest common denominator). **b)** The third line is incorrect. In the previous line, the denominators and the 10 were eliminated instead of being simplified. The third line should be $2(2y + 4) = 5(y - 3)$.

10. 18 cm

11. **a)** 86°F **b)** 25°C

12. **a)** $a = \frac{3}{2}$ **b)** $u = -\frac{18}{11}$ **c)** $w = \frac{57}{29}$