Solving Equations—Quick Reference

Integer Rules

Addition:

- If the signs are the same, add the numbers and keep the sign.
- If the signs are different, subtract the numbers and keep the sign of the number with the largest absolute value.

Subtraction: Add the opposite

Keep—Change—Change

- Keep the first number the same.
- Change the subtraction sign to addition.
- Change the sign of the second number to it's opposite sign.

Multiplication and Division:

- If the signs are the same, the answer is positive.
- If the signs are different, the answer is negative.

Golden Rule for Solving Equations:

Whatever You Do To One Side of the Equation, You Must Do to the Other Side!

Combining Like Terms

Like terms are two or more terms that contain the same **variable.**

Example:	3x, 8x, 9x	are like terms.
	2y, 9y, 10y	are like terms.
	3x, 3y ↑ ↑	are **NOT** like terms because they do **NOT** have the same variable!

Distributive Property Examples

$3(x+5) = 3x + 15$	**Multiply the 3 times x and 5.**
$-2(y - 5) = -2y + 10$	**Multiply –2 times y and –5.**
$5(2x - 6) = 10x - 30$	**Multiply 5 times 2x and –6.**

Solving Equations Study Guide

1. Does your equation have fractions?
   Yes—Multiply every term (on both sides) by the denominator.
   No—Go to Step 2.

2. Does your equation involve the distributive property? (Do you see parenthesis?)
   Yes—Rewrite the equation using the distributive property.
   No—Go to Step 3.

3. On either side, do you have like terms?
   Yes—Rewrite the equation with like terms together. Then combine like terms.
   (Don't forget to take the sign in front of each term!)
   No– Go to Step 4.

4. Do you have variables on both sides of the equation?
   Yes—Add or subtract the terms to get all the variables on one side and all the constants on the other side. Then go to step 6.
   No—Go to Step 5.

5. At this point, you should have a basic two-step equation. If not go back and recheck your steps above.
   - Use Addition or Subtraction to remove any constants from the variable side of the equation.
   (Remember the Golden Rule!)

6. Use multiplication or division to remove any coefficients form the variable side of the equation.
   (Remember the Golden Rule!)

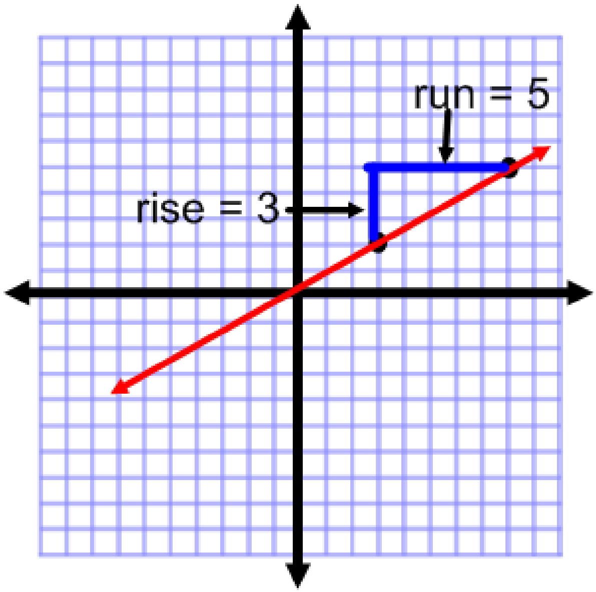
7. Check your answer using substitution!

Congratulations! You are finished with the problem!

Graphing Equations – Quick Reference

Slope= $\frac{\text{rise}}{\text{run}}$

- Calculate the slope by choosing two points on the line.
- Count the rise (how far up or down to get to the next point?) This is the numerator.
- Count the run (how far left or right to get to the next point?) This is the denominator.
- Write the slope as a fraction.



Slope = 3/5

** Read the graph from left to right. If the line is **falling**, then the slope is **negative**.
If the line is **rising**, the slope is **positive**.

When counting the rise and run, if you count **down or **left**, then the number is **negative**. If you count **up** or **right**, the number is **positive**.

Slope Intercept Form

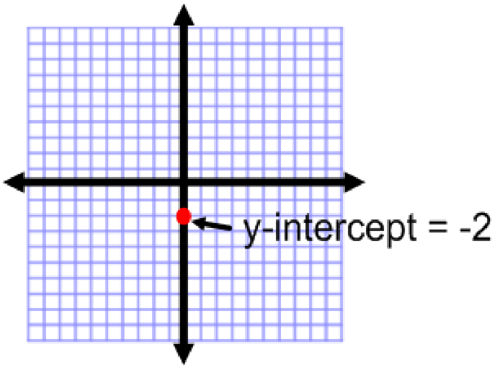
$$y = \underset{\text{Slope}}{\underset{\uparrow}{m}}x + \underset{\text{Y-intercept}}{\underset{\uparrow}{b}}$$

Graphing Using Slope Intercept Form

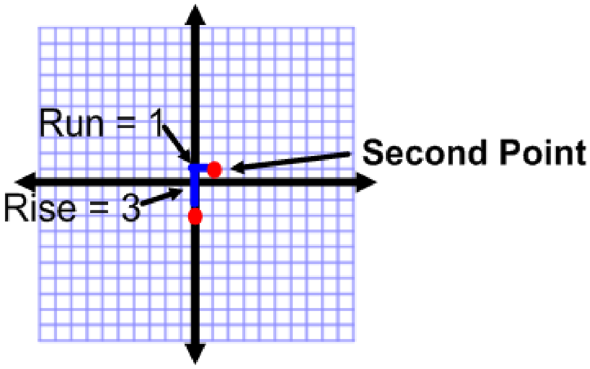
1. Identify the slope and y-intercept in the equation.

$$y = \underset{\text{Slope}}{\underset{\uparrow}{3}}x - \underset{\text{Y-intercept}}{\underset{\uparrow}{2}}$$

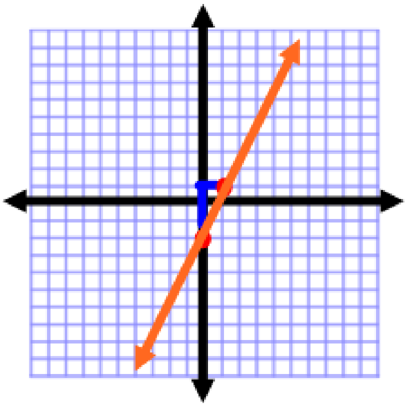
2. Plot the y-intercept on the graph.



3. From the y-intercept, count the rise and run for the slope. Plot the second point.



4. Draw a line through your two points.



Writing Equations—Quick Reference

Slope Intercept Form

$$y = \underset{\underset{\text{Slope}}{\uparrow}}{m}x + \underset{\underset{\text{Y-intercept}}{\uparrow}}{b}$$

If you know the **slope** (or rate) and the **y-intercept** (or constant), then you can easily write an equation in slope intercept form.

Example: If you have a **slope** of **3** and **y-intercept of -4**, the equation can be written as:

$$y = \underset{\underset{\text{slope}}{\uparrow}}{3}x \underset{\underset{\text{y-intercept}}{\uparrow}}{-4}$$

Writing Equations Given Slope and a Point

If you are given slope and a point, then you are given **m, x**, and **y** for the equation

$y = mx + b$.

You must have **slope (m)** and the **y-intercept (b)** in order to write an equation.

Step 1: Substitute m, x, y into the equation and solve for b.

Step 2: Use m and b to write your equation in slope intercept form.

Example: Write an equation for the line that has a slope of 2 and passes through the point (3,1).

m = 2, x = 3 y = 1

y = mx + b

1 = 2(3) + b Substitute for m, x, and y.

1 = 6 +b Simplify (2•3 =6)

1-6 = 6-6- +b Subtract 6 from both sides.

-5 = b Simplify (1-6= -5)

y = 2x -5 Write your equation.

Writing an Equation Given Two Points

If you are given two points and asked to write an equation, you will have to find the slope and the y-intercept!

Step 1: Find the **slope** using: $\frac{y_2 - y_1}{x_2 - x_1}$

Step 2: Use the slope (from step 1) and **one** of the points to find the **y-intercept**.

Step 3: Write your equation using the **slope** (step 1) and **y-intercept** (step 2).

Example: Write an equation for the line that passes through **(1,6) (3,-4).**

Step 1: $\frac{-4-6}{3-1} = \frac{-10}{2} = -5$ **Slope = -5**

Step 2: y = mx +b m = -5 (1,6)

y = mx + b

6 = -5(1) +b

6 = -5 +b Simplify: -5(1)= -5.

6 +5= -5 +5+b Add 5 to BOTH sides.

11 = b Simplify (6+5=11).

Y-intercept = 11

Step 3: y = -5x+ 11

Standard Form

$$Ax + By = C$$

The trick with standard form is that **A, B**, and **C** must be **integers** AND **A** must be a **positive integer**!

Examples:

-3x + 2y = 9 Incorrect! -3 must be positive (multiply all terms by -1)

3x – 2y = -9 Correct! A, B, & C are integers and A is a positive integer.

Systems of Equations – Quick Reference

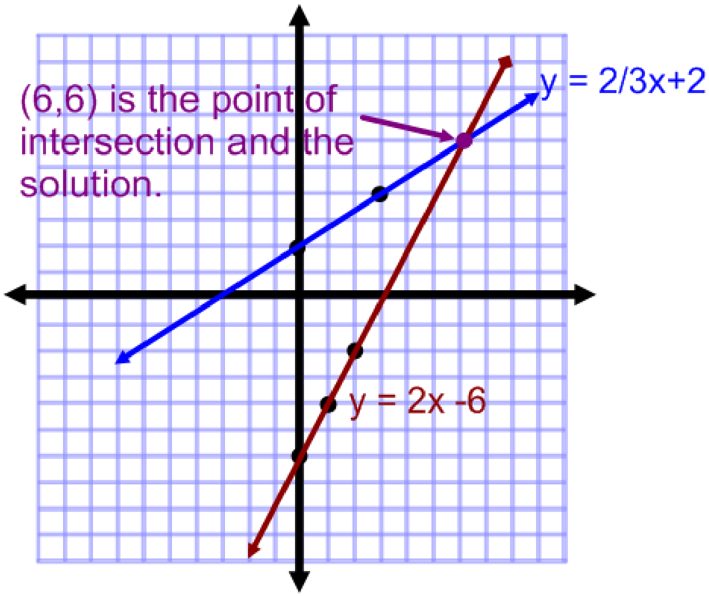
Two linear equations form a system of equations. You can solve a system of equations using one of three methods:

1. **Graphing**
2. **Substitution Method**
3. **Linear Combinations Method**

Graphing Systems of Equations

$y = 2/3x + 2$

$y = 2x - 6$



The solution to this system of equations is (6,6)

The solution to a system of equations is the **point of intersection**.

The **ordered pair** that is the point of intersection represents the solution that satisfies **BOTH** equations.

If two lines are **parallel** to each other, then there is **no solution**. The lines will never intersect.

If two lines lay **one on top of another** then there are **infinite solutions**. Every point on the line is a solution.

Substitution Method

Solve the following system of equations:

$$\mathbf{x - 2y = -10}$$
$$\mathbf{y = 3x}$$

$x - 2y = -10$ $x - 2(3x) = -10$	Since we know y = 3x, substitute 3x for y into the first equation.
$x - 6x = -10$	Simplify: Multiply 2(3x) = 6x.
$-5x = -10$	Simplify: x – 6x = -5x
$\frac{-5x}{-5} = \frac{-10}{-5}$	Solve for x by dividing both sides by -5.
$x = 2$	The x coordinate is 2.
$y = 3x$ $y = 3(2)$ **$y = 6$**	Since we know that x = 2, we can substitute 2 for x into y = 3x.
Solution: (2, 6)	The solution!

Linear Combinations (Addition Method)

Solve the following system of equations:

$$\mathbf{3x + 2y = 10}$$
$$\mathbf{2x + 5y = 3}$$

$-2(3x + 2y = 10)$ $3(2x + 5y = 3)$	Create opposite terms. I'm creating opposite x terms.
$-6x - 4y = -20$ $\underline{6x + 15y = 9}$ $11y = -11$	Multiply to create opposite terms. Then add the like terms.
$\frac{11y}{11} = \frac{-11}{11}$	Solve for y by dividing both sides by 11.
$y = -1$	The y coordinate is -1
$2x + 5y = 3$ $2x + 5(-1) = 3$	Substitute -1 for y into one of the equations.
$2x - 5 = 3$ $2x - 5 + 5 = 3 + 5$ $\frac{2x}{2} = \frac{8}{2}$	Solve for x!
$x = 4$	**The solution (4, -1)**

Inequalities – Quick Reference

Inequality Symbols

< **Less Than**
≤ **Less Than OR Equal To**

> **Greater Than**
≥ **Greater Than or Equal To**

Graphing Inequalities in One Variable

Graphing Symbols

○⟶ **Greater Than** (The open circle indicates that this is **NOT Equal to** the numeral graphed.

●⟶ **Greater Than or Equal To** (The closed circle indicates that this is **Equal to** the numeral graphed.

⟵○ **Less Than** (The open circle indicates that this is **NOT Equal to** the numeral graphed.

⟵● **Less Than or Equal To** (The closed circle indicates that this is **Equal to** the numeral graphed.

Special Rule - Just for Inequalities

Whenever you **multiply or divide** by a **negative** number, you **MUST reverse** the sign.

Example

$-3x < 9$

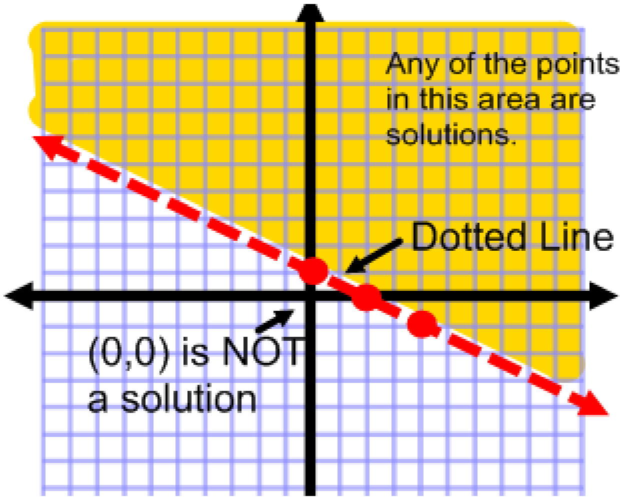
Divide by a negative 3 → $\frac{-3x}{-3} < \frac{9}{-3}$ Reverse the sign

$x > -3$

Graphing Inequalities in Two Variables

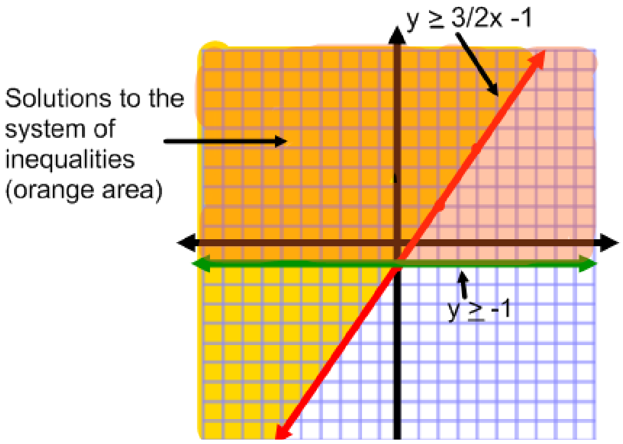
Graph for: **y > -1/2x + 1**

1. Graph y = -1/2x + 1, but dot the line since the symbol is >. The points on the line are **not** solutions.
2. Pick a point such as (0,0) and substitute it into the inequality. (0,0) is **not** a solution, therefore, shade the side of the line that does not contain (0,0).



Systems of Inequalities

Graph each inequality as shown above. **ONLY** the area that is shaded by **BOTH** inequalities is the solution set (orange section)



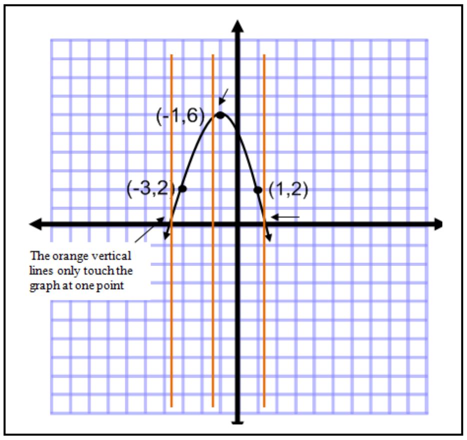
Functions – Quick Reference

Function Notation can be written as:

$f(x) = 3x+2$ this translates to: "f of x" equals 3x+2"
$g(x) = 3x-1$ this translates to: "g of x equals 3x – 1"

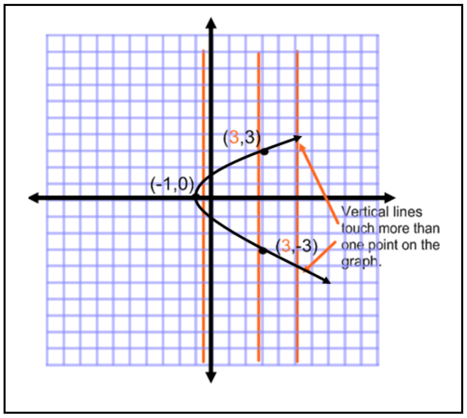
Identifying Functions using the Vertical Line Test

If a graph represents a function, that graph will only intersect with a vertical line one time.



When vertical lines are drawn through this graph, each vertical line touches the graph only one time.

This graph represents a function.



When vertical lines are drawn through this graph, each vertical line touches the graph more than once.

This graph does not represent a function.

Evaluating Functions

f(x) = 6x -1	**Find f(5)**	**Original Problem**
↓ f(x) = 6x -1	↓ Find f(5)	← Notice how 5 replaces the x in the function notation.
f(5) = 6(5) -1	Substitute 5 for x in the original function.	
f(5) = 29	Evaluate! This is your answer!	

This answer means that if you substitute 5 for x, into this function, you will get an answer of 29! You "used" to write: y = 29. Now, in place of y, you will use f(5).

**(The 5 can be replaced with whatever number you substitute into the equation.)

Linear Functions

Function notation can be confusing, but once you can identify the x and y coordinate, you can think of your typical ordered pair.

A typical ordered pair: (2, 5) where $(\underset{\underset{x}{\uparrow}}{2}, \underset{\underset{y}{\uparrow}}{5})$

An ordered pair using **function notation:** $\mathbf{f(\underset{\underset{x}{\uparrow}}{2}) = \underset{\underset{y}{\uparrow}}{5}}$

f(x) = y coordinate

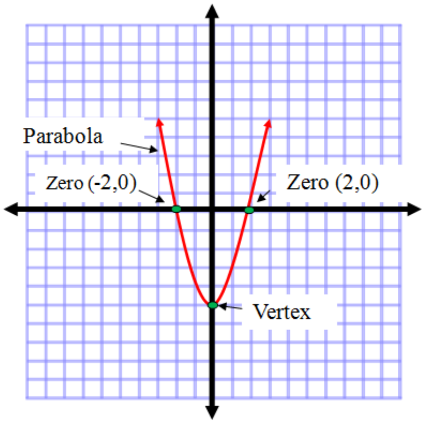
Quadratic Functions

Quadratic Functions will have a "squared term"

$f(x) = \underset{\uparrow}{a}x^2 + \underset{\nearrow}{b}x + \underset{\nwarrow}{c}$ coefficients constant

$f(x) = \underset{\uparrow}{2}x^2 + \underset{\nearrow}{3}x + \underset{\uparrow}{4}$ coefficients constant

A quadratic function will result in a "parabola" when graphed.



**If the lead coefficient is positive, then the parabola will open up. Example: $3x^2 + 2x - 5$ (3 is positive)
**If the lead coefficient is negative, then the parabola will open down. Example: $-2x^2 + 2x - 5$ (2 is negative)

Vertex Formula

Given the function: $f(x) = ax^2 + bx + c$

Vertex Formula: $\frac{-b}{2a}$ (The opposite of b divided by 2 times a)

Exponents and Monomials—Quick Reference

4^2	This expression is read as "4 to the second power" OR "4 squared".
$4^2 = 4 \cdot 4$	It means that we multiply 4 by itself 2 times.
$4^2 = 16$	$4 \cdot 4 = 16$

Tip!

Whenever you have a **negative base** and the **exponent** is **even**, your answer will always be **positive!**

Whenever you have a **negative base** and the **exponent** is **odd**, your answer will always be **negative!**

$(-3)^3$	This expression is read as -3 to the third power.
$(-3)^3 = -3 \cdot -3 \cdot -3$	It means that we multiply -3 by itself 3 times.
$(-3)^3 = -27$	$\underbrace{-3 \cdot -3}_{9} \cdot -3 = -27$ $9 \cdot (-3) = -27$

LAWS of EXPONENTS

Multiplying Powers with the Same Base

Property: When multiplying powers with the <u>same</u> base, **add the exponents.**

$y^3 \cdot y^4 = \mathbf{y^7}$ Since the bases are the same (y), you can add the exponents: 3+4 = 7.

Power of a Power Property

Property: To find the <u>power of a power</u>, **multiply the exponents.**

$(a^3)^5 = \mathbf{a^{15}}$ Multiply the exponents.

Power of a Product Property

Property: To find the power of a product, **find the power of each factor and multiply.**

Think of it as distributing the exponent to each factor!

$(2xy)^3 = 2^3x^3y^3 = \mathbf{8\,x^3y^3}$

$\downarrow \;\; \downarrow\downarrow$

$8\,x^3y^3$

$2^3 = 8$. x^3y^3 cannot be combined because the bases are not the same.

Power of Quotient Property

Property: To find the power of a quotient, **raise the numerator to the power, and the denominator to the power. Then divide.**

$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$$

Zero Exponents

Any number (except 0) to the zero power is equal to 1.

$4^0 = 1$ $10^0 = 1$ $22^0 = 1$ $y^0 = 1$

The Rule for Negative Exponents:

The expression a^{-n} is the reciprocal of a^n

$$3x^{-2} = \frac{3}{x^2}$$

**In this problem, only the x contains the negative exponent, so we only take the reciprocal of x^2.

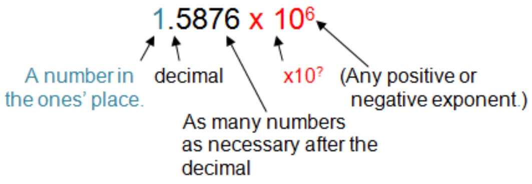
Multiplying Monomials Example

$(3x^2y^3z)^2\,(-3xy^4z)$	Original Problem
$(3x^2y^3z)^2\,(-3xy^4z)$ $\downarrow$ $(9x^4y^6z^2)\,(-3xy^4z)$	The first monomial is raised to the second power. Every constant and variable must be raised to the second power. **The second monomial is not raised to a power, so leave it as is!
$(9x^4y^6z^2)\,(-3xy^4z) = -27$	Multiply your coefficients.
$(9x^4y^6z^2)\,(-3xy^4z) = -27x^5y^{10}z^3$	Multiply the variables with like bases. (Add the exponents.)
$(3x^2y^3z)^2\,(-3xy^4z) = \mathbf{-27x^5y^{10}z^3}$	Final Answer.

Simplifying Monomials Example

$\frac{2x^2y^3}{3x} \cdot \frac{9x^2y^2}{y^4} =$		Original Problem
$\frac{2x^2y^3}{3x} \cdot \frac{9x^2y^2}{y^4} =$	$\frac{18x^4y^5}{\square}$	Step 1: Multiply the numerators. Add the exponents of like bases.
$\frac{2x^2y^3}{3x} \cdot \frac{9x^2y^2}{y^4} =$	$\frac{18x^4y^5}{3xy^4}$	Step 2: Multiply the denominators. **There are no like bases, so we can't add the exponents.
$\frac{18x^4y^5}{3xy^4} =$	$\frac{6}{\square}$	Step 3: Divide the coefficients, if possible.
$\frac{18x^4y^5}{3xy^4} =$	$\frac{6x^3y}{\square}$	Step 4: Subtract the exponents of like bases. $\frac{x^4}{x} = x^3$ and $\frac{y^5}{y^4} = y$
$\frac{2x^2y^3}{3x} \cdot \frac{9x^2y^2}{y^4} =$	$6x^3y$	Final Answer!

Scientific notation must always be written with the same components as the following model:



Polynomials – Quick Reference

What is a Polynomial?

Polynomials can also be classified according to the number of terms. Let's take a look!

$2x$	Monomial	Monomials consist of 1 term
$2x + 3y$ ↑ ↑ 1 2	Binomial	Binomials consist of 2 terms
$2x^2 + 3x + 5$ ↑ ↑ ↑ 1 2 3	Trinomial	Trinomials consist of 3 terms.
$3x^3 + 2x^2 - 6x + 2$ ↑ ↑ ↑ ↑ 1 2 3 4	Polynomial	If there are more than 3 terms, use the term polynomial.

What is the Degree of a Polynomial?

Let's take a look at one more definition! The **degree** of a polynomial **with one variable** is the **highest power** to which the variable is raised. Take a look!

Degree of Polynomials

$6x^3 - 2x^2 + 2x - 1$ ↑ Largest power is 3	A polynomial of degree 3
$2x - 9$ ↑ **When there is no exponent, it is assumed to be 1; therefore this is a degree of 1.	A binomial of degree 1
$-8x^5$ ↑ The exponent is 5	A monomial of degree 5

Adding Polynomials

You must remember that you can only add terms that are **like terms.**

$(3a^4 + 2a^3 - 2a^2 + a + 5) + (4a^4 - a^3 + 5a^2 - 2a - 4)$

$3a^4 + 4a^4 + 2a^3 - a^3 - 2a^2 + 5a^2 + a - 2a + 5 - 4$ Rewrite with like terms together.

$7a^4 + a^3 + 3a^2 - a + 1$ Combine like terms.

Solution:
$7a^4 + a^3 + 3a^2 - a + 1$ **This is the solution.**

F irst Terms
O utside Terms
I nside Terms
L ast Terms

Squaring a Binomial

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

Subtracting Polynomials

You must remember to use Keep Change Change.

If you have a **subtraction** sign **preceding** a set of **parenthesis**, then you must rewrite the problem as an addition problem. We are going to **ADD the OPPOSITE**

Subtraction sign & parenthesis

$(2x - 6) - (3x^2 + 2x - 6)$ Rewritten as: **$(2x - 6) + (-3x^2 - 2x + 6)$**

$(2x - 6)$	**–**	**$(3x^2 + 2x - 6)$**
↑	↑	↑
Keep the same	Change to Addition	Change the sign of every term
↓	↓	↓ ↓ ↓
$(2x - 6)$	**+**	**$-3x^2 - 2x + 6$**

**You must change the sign of every term (to its' opposite sign) inside the set of parenthesis that follows the subtraction sign.

Multiplying Polynomials

We must use our laws of exponents in order to multiply polynomials.

$2a^2b^2 (a^3 + 3ab - b^3)$ **Original Problem**

$2a^2b^2(a^3) + 2a^2b^2(3ab) + 2a^2b^2(-b^3)$ Distribute $2a^2b^2$ througho the parenthesis.

$2a^5b^2 \quad + 6a^3b^3 \quad - 2a^2b^5$ Multiply the coefficients and add the exponents of like bases for each term.

Solution:
$2a^5b^2 + 6a^3b^3 - 2a^2b^5$

Using FOIL

$(3x - 4)(2x + 1)$ Original Problem

$(3x - 4)(2x + 1)$ Multiply the First terms:
$(3x)(2x) = 6x^2$

$6x^2$

$(3x - 4)(2x + 1)$ Multiply the Outside terms:
$(3x)(1) = 3x$

$6x^2 + 3x$

$(3x - 4)(2x + 1)$ Multiply the Inside terms:
$(-4)(2x) = -8x$

$6x^2 + 3x - 8x$

$(3x - 4)(2x + 1)$ Multiply the Last terms:
$(-4)(1) = -4$

$6x^2 + 3x - 8x - 4$

$6x^2 - 5x - 4$ Combine like terms:
$3x - 8x = -5x$
*Notice how this step is the same as the 4th step of Examp 1.

$6x^2 - 5x - 4$ Solution.

Factoring – Quick Reference

Finding the GCF

Find the GCF for: $30x^3 + 5x^2 – 25x$

Step 1: Look at the coefficients

Ask yourself: Is there a number that I can divide 30, 5, and 25 by evenly? Yes, 5!

(30/5 = 6) (5/5=1) (-25/5 = -5)

Step 2: Look at the variable(s).

Ask yourself: Can I factor out a variable from EVERY term?

Yes! Each term has at least one x, therefore, I can factor out x.

Step 3: Identify the GCF:

The GCF for this polynomial is: 5x. (You can divide every term by 5x evenly (without creating a fraction).

Factoring Using the GCF

Factor the greatest common factor from:

$3x^4y^3 + 12x^3y – 18x^2y^2$

Solution:

Step 1: Look at the coefficients.

What is the GCF for 3, 12, 18?

$3x^4y^3 + 12x^3y – 18x^2y^2$

(What is the greatest number that can be divided into all evenly?)

3 is the GCF (for the coefficients).

Step 2: Look at the variable.

Can I factor out a variable for EVERY term?

Yes! Each term contains at least one x^2 and y.(x^2y)

$3x^4y^3 + 12x^3y – 18x^2y^2$

Step 3: Identify the GCF.

The GCF is $3x^2y$. Now we are going to divide EVERY term by $3x^2y$. (Most students do this mentally, but I am going to write it out to show you the process.)

$$\frac{3x^4y^3}{3x^2y} + \frac{12x^3y}{3x^2y} – \frac{18x^2y^2}{3x^2y}$$

$x^2y^2 + 4x – 6y$ (the result after dividing)

Step 4: Write appropriately in factored form.

$$\underset{\substack{\uparrow\\ \text{GCF}}}{3x^2y}\ (\underset{\substack{\uparrow\\ \text{Result after dividing each term by the GCF}}}{x^2y^2} + 4x – 6y)$$

Factored Form: $3x^2y\ (x^2y^2 + 4x – 6y)$

Factoring by Grouping

Factor the following polynomial:

$x^3y^3 + 2x^3 + 4x^2y^3 + 8x^2$

Step 1: **Separate** the polynomial into **2 or more groups** according to common factors. **Identify the common factor** for each group. (In this problem I need to rewrite the problem with common factors side by side.)

$$\underbrace{x^3y^3 + 4x^2y^3}_{\substack{x^2y^3 \text{ is a}\\ \text{common}\\ \text{factor}}}\quad \underbrace{+ 2x^3 + 8x^2}_{\substack{2x^2 \text{ is a}\\ \text{common}\\ \text{factor}}}$$

Step 2: **Divide** each term by the common factor.

$$\frac{x^3y^3}{x^2y^3} + \frac{4x^2y^3}{x^2y^3} + \frac{2x^3}{2x^2} + \frac{8x^2}{2x^2}$$

x + 4 + x + 4 ← Result after dividing

Step 3: Write appropriately in factored form.

$$\underset{\substack{\uparrow\\ \text{Common}\\ \text{Factor}}}{x^2y^3}\ \underset{\substack{\uparrow\\ \text{Result}\\ \text{after dividing}}}{(x+4)} + \underset{\substack{\uparrow\\ \text{Common}\\ \text{factor}}}{2x^2}\ \underset{\substack{\nwarrow\\ \text{Result}\\ \text{after dividing}}}{(x+4)}$$

$$x^2y^3\,(\underset{\uparrow}{x}+4) + 2x^2\,(\underset{\nearrow}{x}+4)$$

Step 4: Now these two terms have a common factor. The common factor is (x+4). We can factor (x+4) and we are left with:

$(x+4)\ (x^2y^3 + 2x^2)$ This is the final answer in factored form.

Factoring Trinomials

Factor: $x^2 - 10x + 21$

Step 1: We are going to form two binomials, so write two sets of parenthesis.

( ) ( )

Step 2: What can we multiply together to get x^2? (The first term of each binomial is multiplied together to get the first term of the trinomial. ($x \cdot x = x^2$)

$$\underset{\substack{\uparrow\\ (x\quad)\ (x\quad)}}{x^2} - 10x + 21$$

Step 3: We need to find **two numbers** that we can **add** together to get **-10** AND **multiply** together to get **21**. (You must remember to take the sign in front of the term with it; therefore, the middle term is -10).

$$x^2 \underset{\uparrow}{- 10x} \underset{\uparrow}{+ 21}$$

Two numbers that we **add** to get **-10** & **multiply** to get **21**.

****In order to add two numbers together and get a negative number and then multiply the same two numbers and get a positive number, both numbers must be negative.*****

Factors of 21	Sum of Factors
-1 • -21 = 21	-1 +(-21) = -22
-3 • -7 = 21	-3+ (-7) = -10

(There are other factors, but I will stop here since I found the one I am looking for)

Our last terms need to be -3 and -7 since when multiplied together they equal 21 and when added together the sum is -10.

Step 4: Complete the binomials.

(x - 3) (x -7) ← Your Solution

Quadratic Equations – Quick Reference

What is a Quadratic Equation?

$ax^2 + bx + c$

$ax^2 + bx + c = 0$	$2x^2 + 3x + 4 = 0$
coefficients constant	coefficients constant

a and **b** are coefficients and **c** is a constant. The one factor that identifies these expressions as **quadratic** is the exponent 2. The first term must always be **ax^2**, and **a** cannot be 0.

Solving Simple Quadratic Equations

$x^2 - 4 = 77$	Our goal is to get x by itself on the left hand side of the equation. We must get rid of the -4 (first) then the exponent 2.
$x^2 - 4 + 4 = 77 + 4$	Add 4 to both sides of the equation.
$x^2 = 81$	Simplify: 77 + 4 = 81
$\sqrt{x^2} = \pm\sqrt{81}$	Take the square root of both sides. (Remember to use the ± sign.)
$x = \pm 9$	There are 2 solutions. X is equal to positive 9 and negative 9.

If $a^2 = b$, then $a = \sqrt{b}$ or $a = -\sqrt{b}$

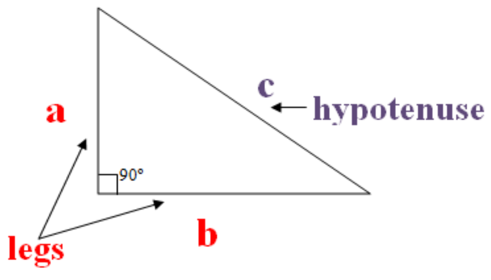
This can also be written as:

$a = \pm\sqrt{b}$

Read as: a = "**plus or minus**" the square root of b.

The Pythagorean Theorem

In any **right triangle**, the sum of the squares of the legs (2 shorter sides) is equal to the square of the hypotenuse (the longest side).



$$a^2 + b^2 = c^2$$

Please Note: This theorem **ONLY** works for **Right Triangles**.

Solving Equations by Factoring

Solve: $x^2 - 7x + 2 = -10$ ←	Our equation is not equal to 0.
$x^2 - 7x + 2 + 10 = -10 + 10$	Before we can factor, we must set our equation equal to 0. Add 10 to both sides.
$x^2 - 7x + 12 = 0$	Now our equation is equal to 0. I can factor.
$(x - 4)(x - 3) = 0$	Factor: $x^2 - 7x + 12$
$x - 4 = 0$ or $x - 3 = 0$	Set both factors equal to 0. (The zero-factor property)
$x = 4$ or $x = 3$	
Check: $x^2 - 7x + 2 = -10$ $4^2 - 7(4) + 2 = -10$ $-10 = -10$ ☺	Substitute the two solutions into the original equation. 4 works! When I substituted I got an answer of -10.
$x^2 - 7x + 2 = -10$ $3^2 - 7(3) + 2 = -10$ $-10 = -10$ ☺	3 works! When I substituted I got an answer of -10.

The Quadratic Formula

Given any quadratic equation:

$$ax^2 + bx + c = 0$$

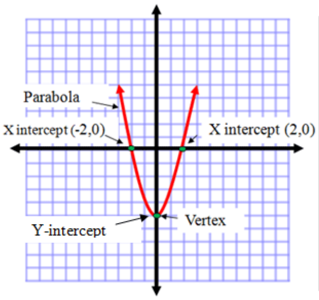
We can substitute the values for a, b, & c into the following formula and solve.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For any quadratic equation in the form:

$y = ax^2 + bx + c$

The graph will result in a parabola.



This parabola opens up and can be classified as **concave up.**

All parabolas that **open up** will have a **positive "a"** value.

The vertex is the lowest point or the **minimum point.**

This parabola opens down and can be classified as **concave down.**

All parabolas that **open down** will have a **negative "a"** value.

The vertex is the highest point or the **maximum point**.