Honors Algebra 1

Unit 4:
Modeling and Analyzing Exponential Functions

Name:____

Fall 2019
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Graphing Exponential Functions

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x)\quad k$, $k\,f(x)$, $f(kx)$, and $f(x\quad k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

F.IF.7e Graph exponential functions, showing intercepts and end behavior.

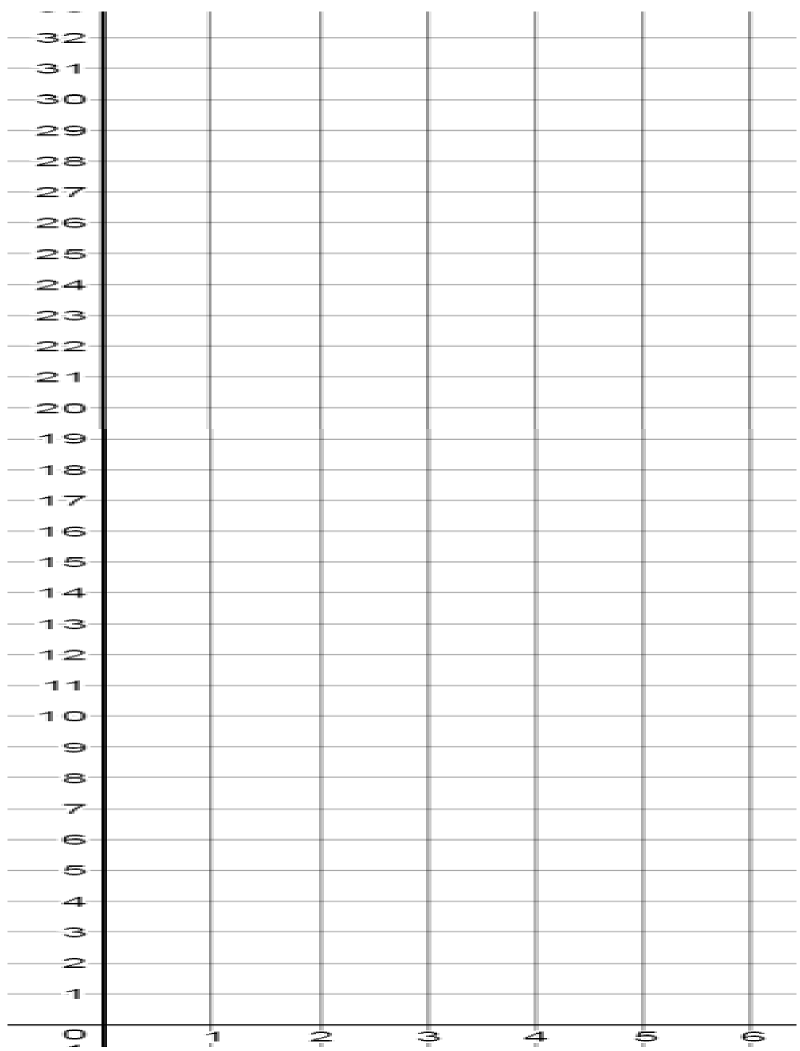
F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

Introduction into exponential functions

Functions are used to model real world situations so we can predict what might happen. For example, a **linear function** can be used to model how much money you can make per hour and a **quadratic function** can model the distance and time of an object when it is thrown due to gravity. **Exponential functions** can be used to model situations where the amount changes rapidly.

Imagine this situation. Ava was going out for a walk and was attacked by some kind of animal. When she inspected her wounds she thought that it was just a scratch but little did she know that she was just infected with a virus that would turn her into a *zombie*. The next day zombie Ava infected Samia and now Samia was turned into a zombie as well. Each day, a zombie can infect one other person and so even though they are slow the zombie virus quickly spreads. How many days will it take for our class to be completely infected?

Day (x)	# Infected (y)
0	1
1	
2	
3	
4	
5	



How many days until we were all infected?

How fast did this outbreak spread?

Can you predict how many people would be infected in 7 days? How did you arrive at the answer?

How many days would it take for the whole school to be infected? (there are about 1750 students in the school) How did you come up with that answer?

The zombie outbreak can be modeled by an exponential function because the rate of the outbreak changes on each day. For example on day 1- only one person could infect, by day 2, two people could infect, by day three 4 people could infect and so on. This means that exponential functions are **not linear** they have a changing **rate of change**.

Vocabulary:
Domain:

Range:

Asymptote:

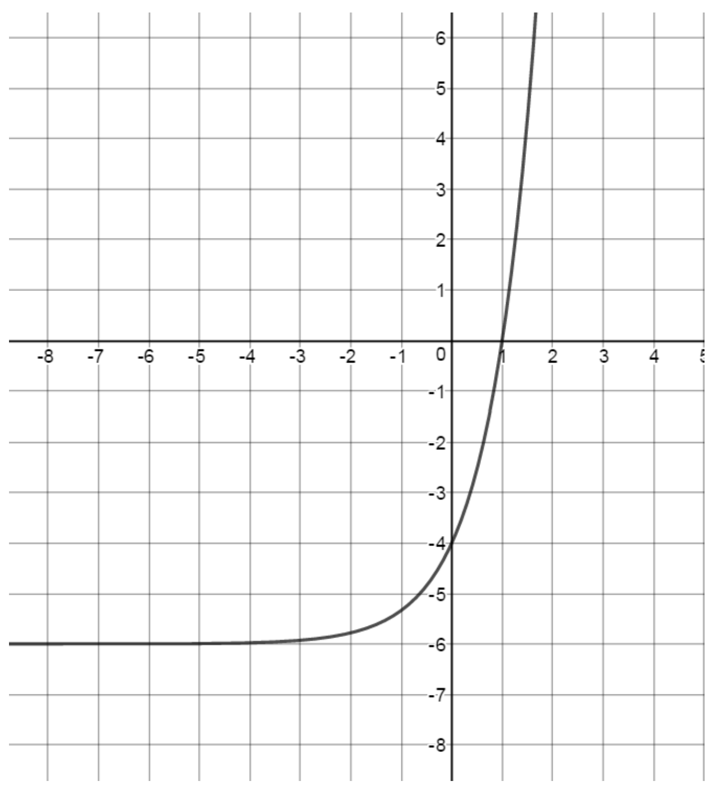
Growth

Decay

End Behavior

Interpreting parts of an exponential graph

Growth/Decay?	
Increasing or decreasing?	
x-intercept	
y-intercept	
Asymptote	
Range	
Left end behavior	
Right end behavior	
Domain	



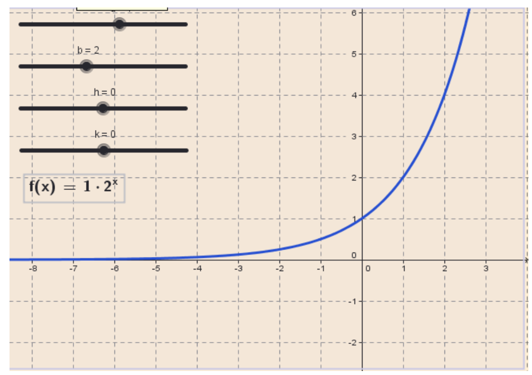
Exponential Transformation Discovery Task

We have discussed the exponential function in class but we have not spent a lot of time with the connection between the function and its corresponding graph. Today you will use GeoGebra to discover how changing different parameters of the exponential function affect the graph.

The base exponential function is the following:

$$y = a(b)^{x-h} + k$$

Go to the blog and open up the GeoGebra worksheet Exponential Transformations



Right now the base function is set as f(x) = 1●2^x

So what is the current value of a? ______

What is the current value of b?_______

What is current value of h?________

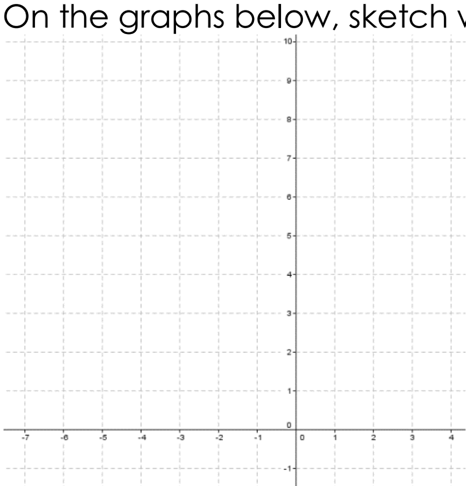
What is the current value of k?______

The Parameter "a"

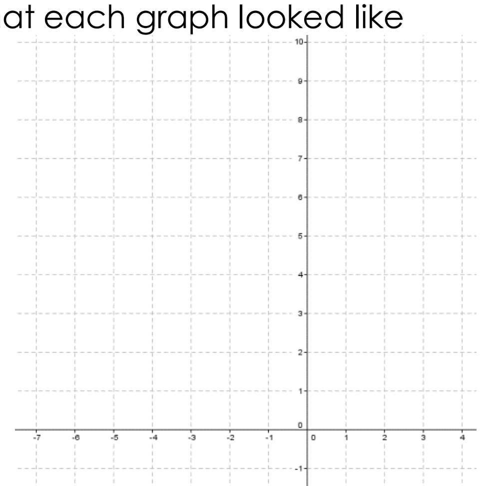
We will begin by investigating the parameter "a". Use the slider to make ***a*** be 2**.** What changes did you see in the graph?

Now change **a** to be 6. What changes do you notice in comparison to its original value of 1?

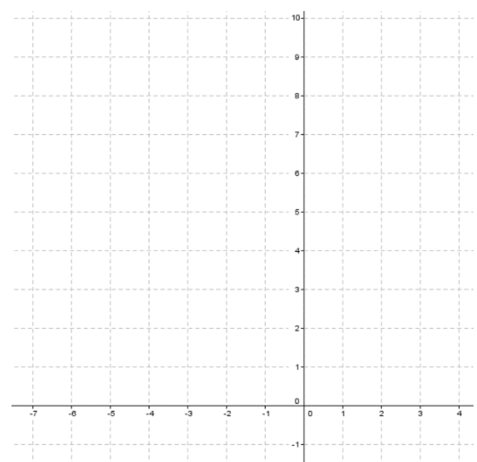
Predict what you think will happen when you change ***a*** to 10. Then move the slider to 10 and see if your prediction was correct.
On the graphs below, sketch what each graph looked like



f(x) = 2 ● 2^x



f(x) = 6 ● 2^x

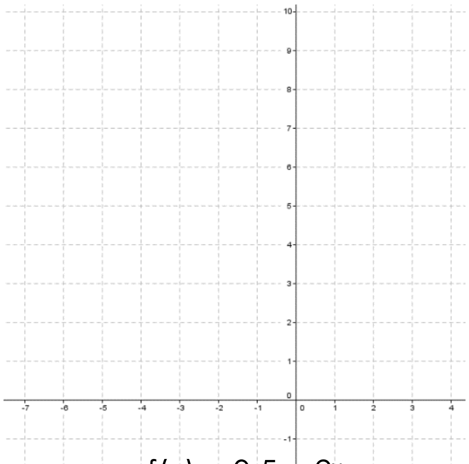


f(x) = 10 ● 2^x

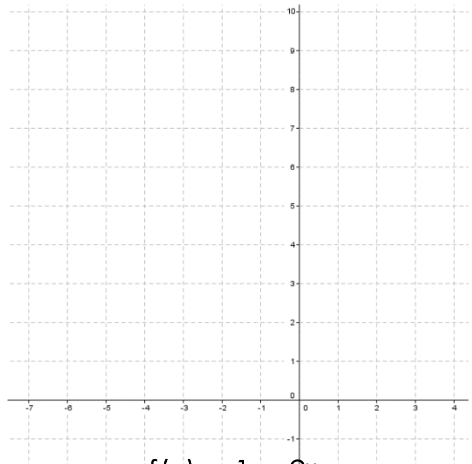
Now let's try when ***a*** is smaller than one. Begin by moving the slider back to the original position of 1. Now move the slider to a = 0.5. What changes did you see?

Now change ***a*** to be 0.1. What changes did you see to the original value of 1?

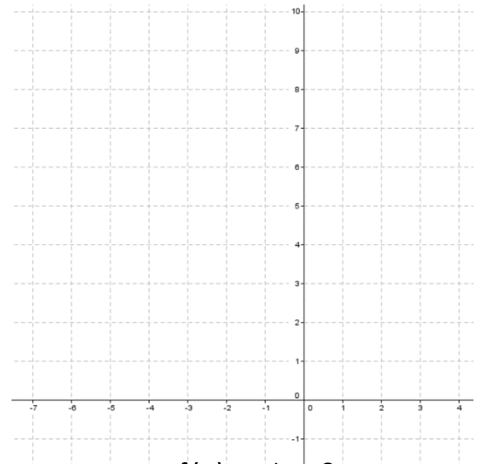
Sketch the following graphs.



$f(x) = 0.5 \bullet 2^x$



$f(x) = 1 \bullet 2^x$



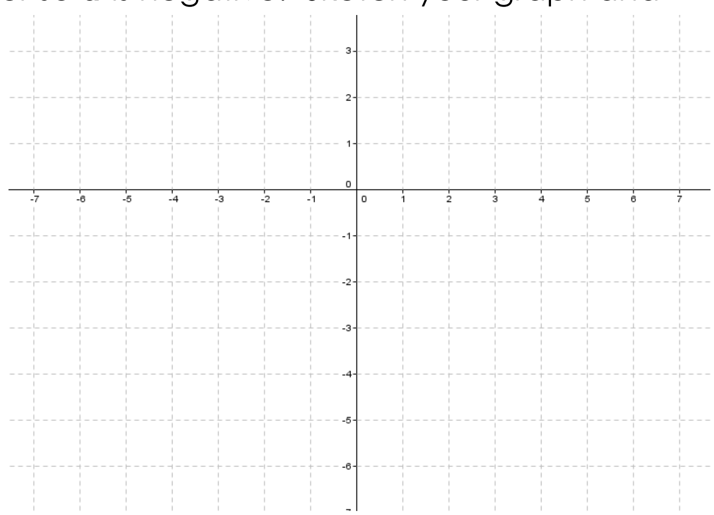
$f(x) = 6 \bullet 2^x$

Make a hypothesis about the value of ***a*** when $0 < a < 1$

Make a hypothesis about the value of ***a*** when $a > 1$

What about when a is negative? Move your slider so ***a*** is negative. Sketch your graph and write your function below.

$f(x) =$ _____ $\bullet\ 2^x$

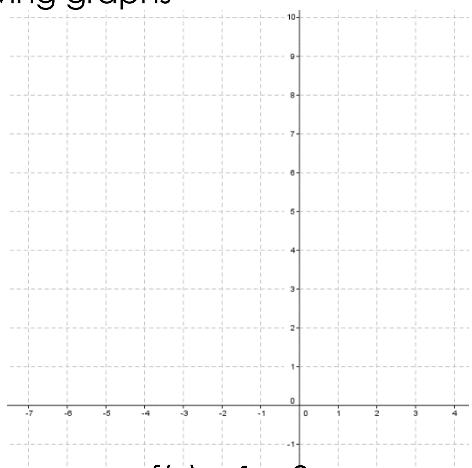


What happens when ***a*** is a negative number?

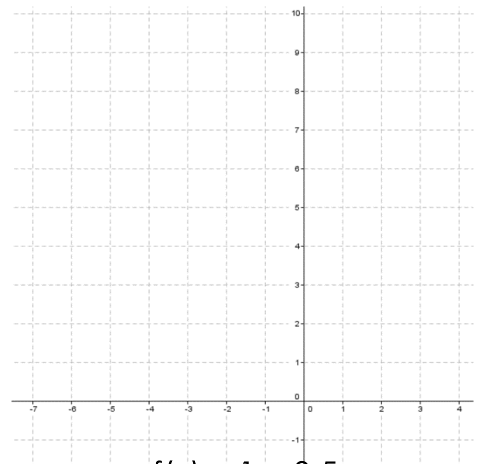
The Parameter “b”

Now we will investigate a new parameter. Move your slider back to the original function f(x) = 1●2^x. Now move the ***b*** slider to 0.5. What change did you notice in the graph?

Sketch the following graphs



f(x) = 1 ● 2^x



f(x) = 1 ● 0.5^x

Make a hypothesis about the value of ***b*** when b > 1

Make a hypothesis about the value of ***b*** when 0 < b < 1

The Parameter “h”

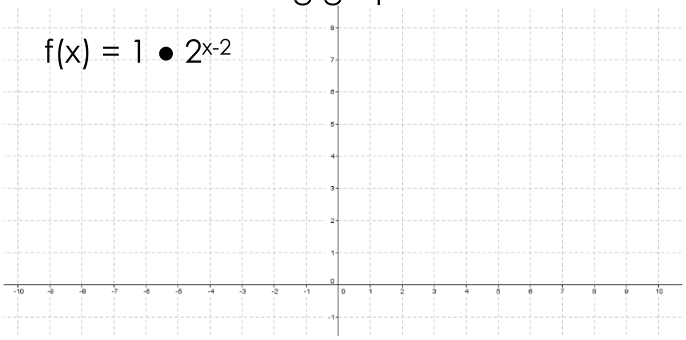
Now we will investigate a new parameter. Move your slider back to the original function f(x) = 1●2^x. Now move the ***h*** slider to 2. What change did you notice in the graph?

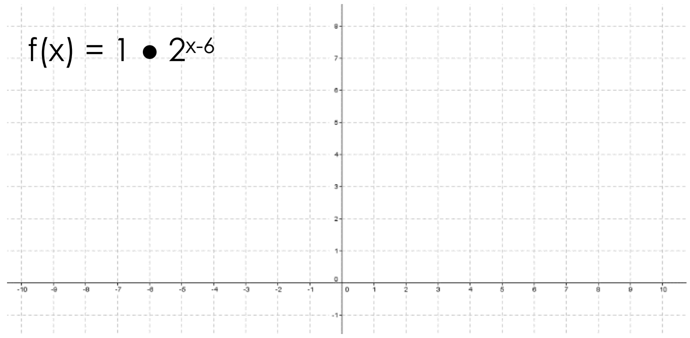
Now change **h** to be 6. What changes do you notice in comparison to its original value of 0?

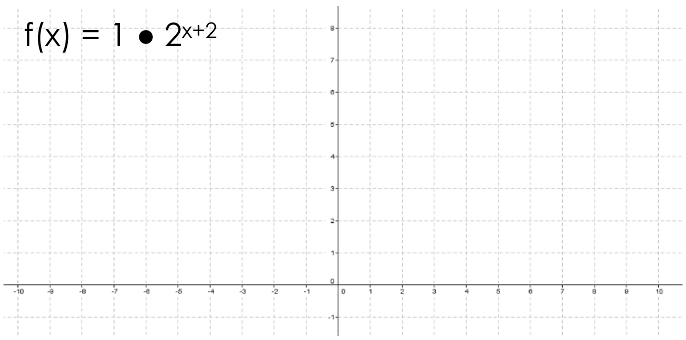
Now change **h** to be -2. What changes do you notice in comparison to its original value of 0?

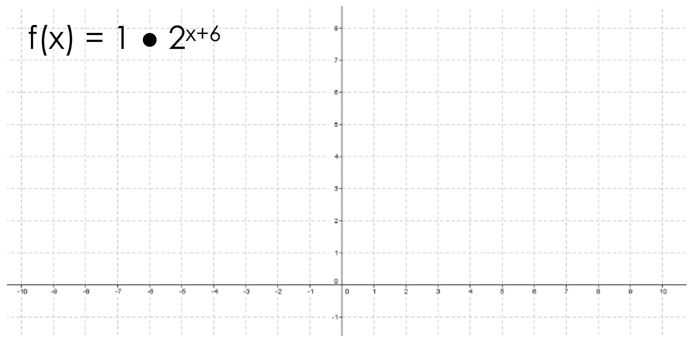
Now change **h** to be -6. What changes do you notice in comparison to its original value of 0?

Sketch the following graphs.









Remember that the base exponential function has x MINUS h in the exponent. With that in mind answer the following.

I hypothesize that when the h is positive the exponential graph…

I hypothesize that when the h is negative the exponential graph…

The Parameter "k"

Now we will investigate a new parameter. Move your slider back to the original function $f(x) = 1 \bullet 2^x$. Now move the ***k*** slider to 2. What change did you notice in the graph?

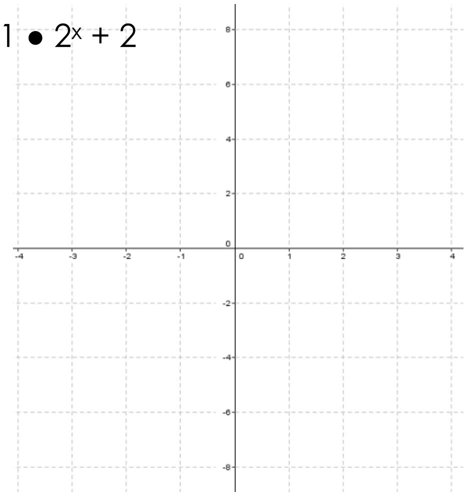
Now change **k** to be 6. What changes do you notice in comparison to its original value of 0?

Now change **k** to be -2. What changes do you notice in comparison to its original value of 0?

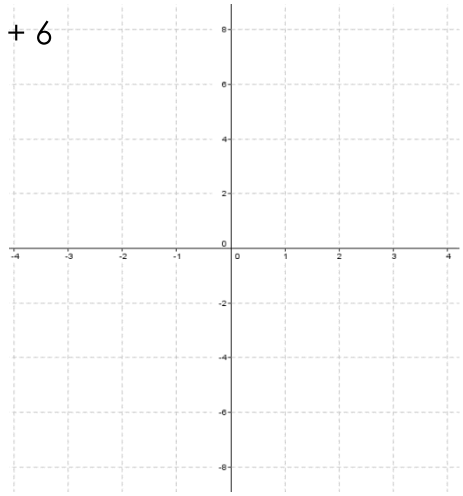
Now change **k** to be -6. What changes do you notice in comparison to its original value of 0?

Sketch the following graphs.

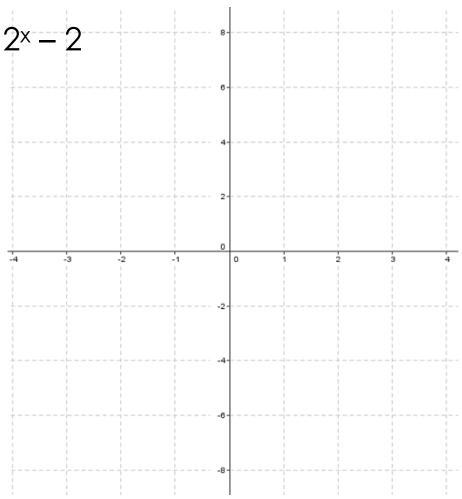
$f(x) = 1 \bullet 2^x + 2$



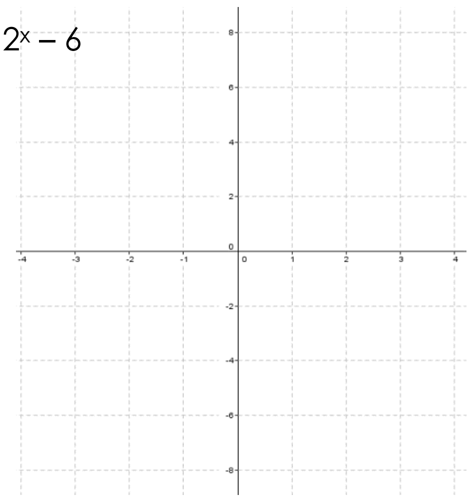
$f(x) = 1 \bullet 2^x + 6$



$f(x) = 1 \bullet 2^x - 2$



$f(x) = 1 \bullet 2^x - 6$



I hypothesize that when the k is positive the exponential graph…

I hypothesize that when the k is negative the exponential graph…

PUTTING THE PIECES TOGETHER

Now that you have discovered how different parameters affect the exponential graph and made hypothesis, let's use the 'math lingo' to correctly match the parameter to its effect. Copy the boxes from the bottom of this paper onto the correct parameter.

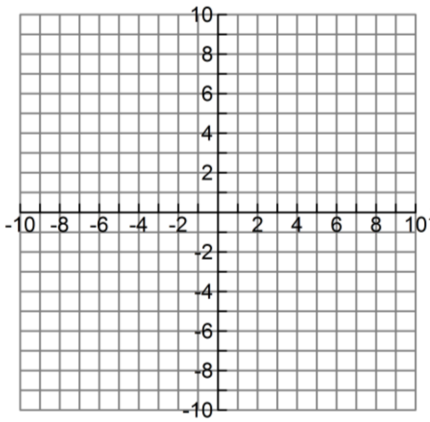
$$y = a(b)^{x-h} + k$$

Parameter	Effect
a	
b	
h	
k	

When it is between 0 and 1 the graph **shrinks** When it is greater than 1 the graph **stretches** When it is **negative** the graph **Reflects over the x-axis**	When it is between 0 and 1 the graph is a **decay** When it is greater than 1 the graph is a **growth**
A **positive** value moves the graph to the **right** A **negative** value moves the graph to the **left**	A **positive** value moves the graph **up** A **negative** value moves the graph **down**

For each of the following ***describe*** how the exponential graph will move from the parent function f(x) = 1●2ˣ. Then sketch the graph. Check your solution by using the GeoGebra application.

$y = \frac{2}{3}(3)^{x-4} - 2$



Growth/Decay?

Stretch/Shrink?

Horizontal Shift?

Vertical Shift?

Reflection?

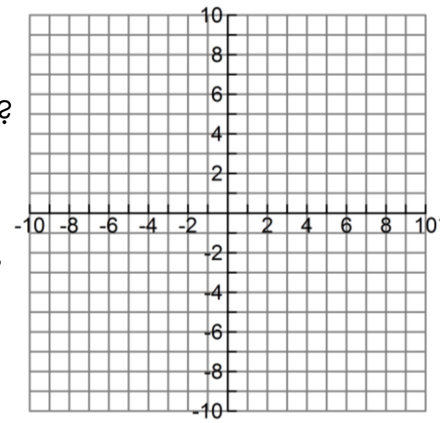
Asymptote: **End Behavior:**

Range:

Domain:

Increase/ decrease?

$y = 5\left(\frac{1}{2}\right)^{x} - 3$



Growth/Decay?

Stretch/Shrink?

Horizontal Shift?

Vertical Shift?

Reflection?

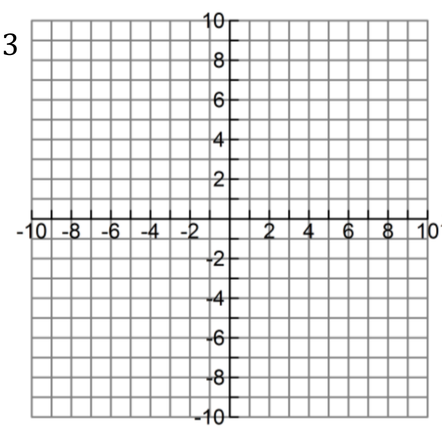
Asymptote: **End Behavior:**

Range:

Domain: **Y-Intercept:**

Increase/ decrease?

$y = -\frac{5}{2}\left(\frac{4}{7}\right)^{x+1} + 3$



Growth/Decay?

Stretch/Shrink?

Horizontal Shift?

Vertical Shift?

Reflection?

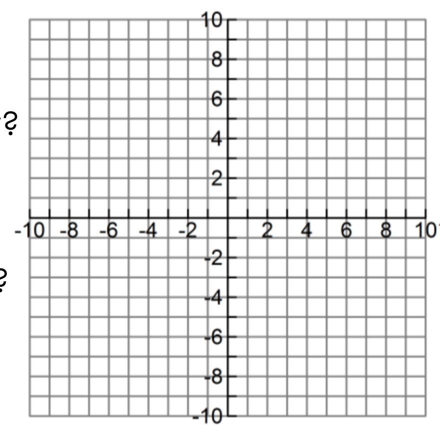
Asymptote: **End Behavior:**

Range:

Domain: **x- intercept:**

Increase/ decrease?

$y = \frac{7}{3}(6)^{x+2}$



Growth/Decay?

Stretch/Shrink?

Horizontal Shift?

Vertical Shift?

Reflection?

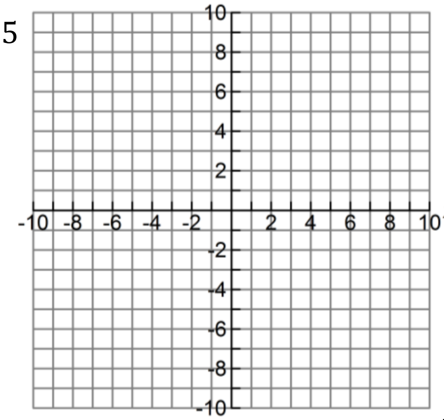
Asymptote: **End Behavior:**

Range:

Domain:

Increase/ decrease?

$y = -\frac{1}{2}\left(\frac{2}{3}\right)^{x-1} + 5$



Growth/Decay?

Stretch/Shrink?

Horizontal Shift?

Vertical Shift?

Reflection?

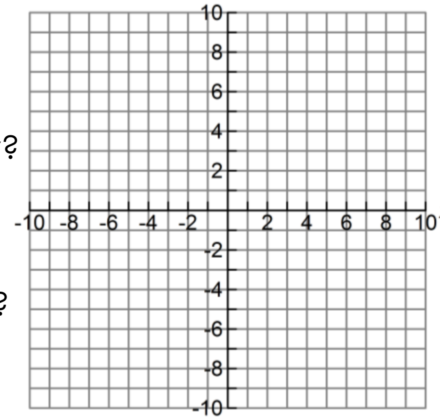
Asymptote: **End Behavior:**

Range:

Domain:

Increase/ decrease?

$y = \left(\frac{8}{3}\right)^{x-3} + 3$



Growth/Decay?

Stretch/Shrink?

Horizontal Shift?

Vertical Shift?

Reflection?

Asymptote: **End Behavior:**

Range:

Domain: **Zeros:**

Increase/ decrease?

EXPONENTIAL FUNCTIONS PRACTICE

A. FILL IN THE BLANK

Greater than 1	left	Parenthesis	stretch	up
Between 0 and 1	shrink	right	down	reflection

1. To determine if a function is growth or decay look at the number inside the ______________________.
2. If the number is _____________________________ it is a growth function.
3. If the number is _____________________________ it is a decay function.
4. To determine if a function is a ____________ see if the number in front is greater than 1
5. To determine if a function is a ____________ see if the number in front is between 0 and 1
6. If there is a negative in front of the equation then there is a ____________________.
7. If there is + in the exponent then there is a _________________ shift.
8. If there is - in the exponent then there is a _________________ shift.
9. $f(x) = 3(2)^{x+1} - 3$ moves ________ by 3
10. $f(x) = 3\left(\frac{1}{2}\right)^{x-1} + 2$ moves ________ by 2

B. Analyze the following functions

11. $y = 3\left(\frac{1}{2}\right)^{x-3} + 1$	12. $y = -2(4)^{x+2} - 3$
Stretch or Shrink?_______________	Stretch or Shrink?_______________
By how much?_________________	By how much?_________________
Growth or Decay?__________________	Growth or Decay?__________________
Reflection or no Reflection?_________	Reflection or no Reflection?_________
Horizontal Shift?_________________	Horizontal Shift?_________________
Vertical Shift?___________________	Vertical Shift?___________________
Asymptote?_____________________	Asymptote?_____________________

13. $y=-2\left(\frac{5}{2}\right)^{x}-5$

Stretch or Shrink?________________

By how much?__________________

Growth or Decay?_______________

Reflection or no Reflection?_______

Horizontal Shift?__________________

Vertical Shift?____________________

Asymptote?______________________

14. $y=\frac{1}{3}(4)^{x+1}$

Stretch or Shrink?________________

By how much?__________________

Growth or Decay?________________

Reflection or no Reflection?_______

Horizontal Shift?__________________

Vertical Shift?____________________

Asymptote?______________________

15. $y=\frac{1}{4}(6)^{x-1}-3$

Stretch or Shrink?________________

By how much?__________________

Growth or Decay?______________

Reflection or no Reflection?_______

Horizontal Shift?__________________

Vertical Shift?____________________

Asymptote?______________________

16. $y=-4\left(\frac{2}{3}\right)^{x}+5$

Stretch or Shrink?________________

By how much?__________________

Growth or Decay?________________

Reflection or no Reflection?______

Horizontal Shift?__________________

Vertical Shift?____________________

Asymptote?______________________

17. $y=\frac{1}{2}\left(\frac{3}{4}\right)^{x-7}$

Stretch or Shrink?________________
By how much?__________________

Growth or Decay?______________

Reflection or no Reflection?_______

Horizontal Shift?__________________

Vertical Shift?____________________

Asymptote?______________________

18. $y=-(4)^{x}+1$

Stretch or Shrink?________________
By how much?__________________

Growth or Decay?_______________

Reflection or no Reflection?_______

Horizontal Shift?__________________

Vertical Shift?____________________

Asymptote?______________________

C. Answer the questions given the graphs below

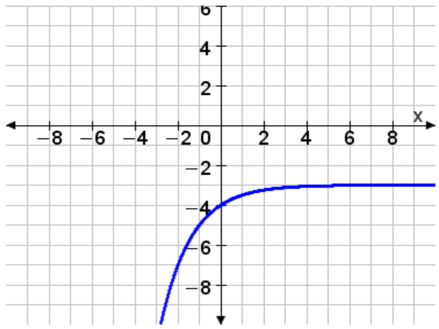
19. Which of the following could be the equation for the graph shown

A. $f(x) = -\left(\frac{1}{2}\right)^{x-3}$

B. $f(x) = \left(\frac{1}{2}\right)^{x} - 3$

C. $f(x) = -(2)^{x} - 3$

D. $f(x) = -\left(\frac{1}{2}\right)^{x} - 3$



20. Domain:

21. Range:

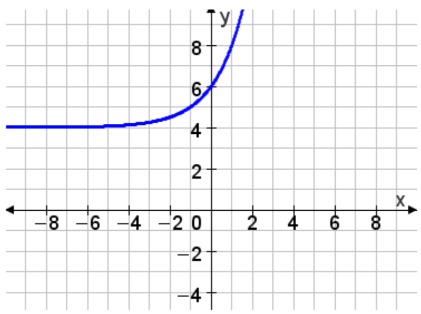
22. Which of the following could be the equation for the graph

A. $f(x) = \frac{3}{4}(2)^{x} - 4$

B. $f(x) = 2\left(\frac{1}{2}\right)^{x} + 4$

C. $f(x) = 2(2)^{x} + 4$

D. $f(x) = 2(2)^{x+4}$



23. Domain:

24. Range:

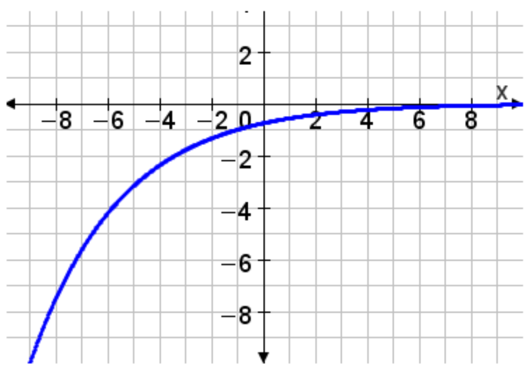
25. Which of the following could be the equation for the graph shown?

A. $f(x) = -\left(\frac{3}{4}\right)^{x+1}$

B. $f(x) = -\left(\frac{3}{4}\right)^{x} + 1$

C. $f(x) = -\left(\frac{4}{3}\right)^{x}$

D. $f(x) = \left(\frac{3}{4}\right)^{x+1}$



26. Domain:

27. Range

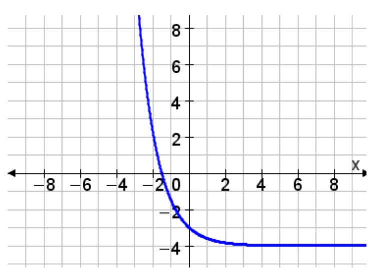
28. Which of the following could be the equation for the graph shown?

A. $f(x) = \left(\frac{5}{2}\right)^{x} - 4$

B. $f(x) = \left(\frac{2}{5}\right)^{x} - 4$

C. $f(x) = -\left(\frac{2}{5}\right)^{x} - 4$

D. $f(x) = -\left(\frac{5}{2}\right)^{x} - 4$

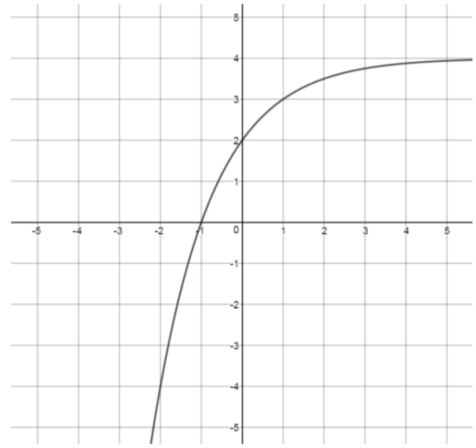


29. Domain:

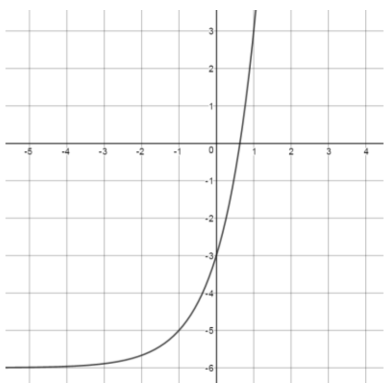
30. Range:

1) Analyze all of the key features of the graph

Domain:__________
Range:__________
Y-intercept:________
X-Intercept:________
Asymptote:_________
Growth or Decay?
Increasing/Decreasing?
End Behavior $As\ x \to -\infty, y \to$ ____
$As\ x \to +\infty, y \to$ ____



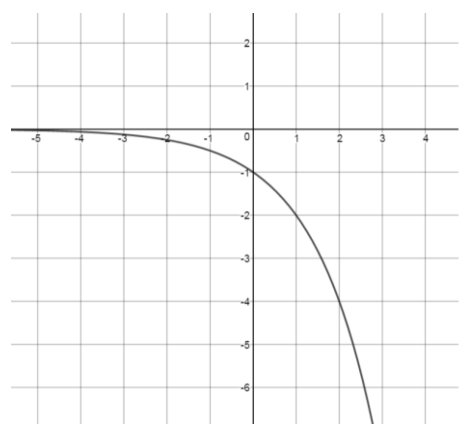
2)



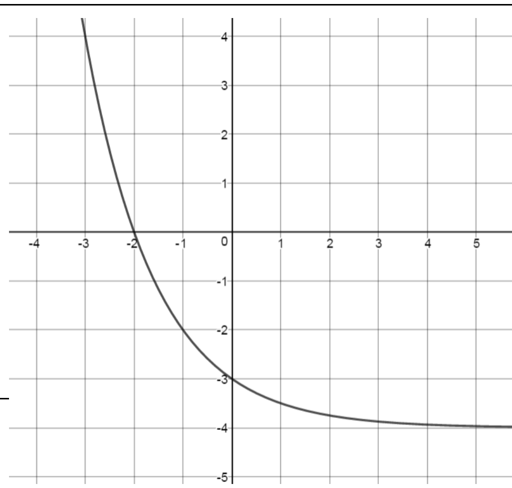
Domain:__________
Range:__________
Y-intercept:________
X-Intercept:________
Asymptote:_________
Growth or Decay?
Increasing/Decreasing?
End Behavior $As\ x \to -\infty, y \to$ ____
$As\ x \to +\infty, y \to$ ____

3)

Domain:__________
Range:__________
Y-intercept:________
X-Intercept:________
Asymptote:_________
Growth or Decay?
Increasing/Decreasing?
End Behavior $As\ x \to -\infty, y \to$ ____
$As\ x \to +\infty, y \to$ ____



4)



Domain:__________
Range:__________
Y-intercept:________
X-Intercept:________
Asymptote:_________
Growth or Decay?
Increasing/Decreasing?
End Behavior $As\ x \to -\infty, y \to$ ____
$As\ x \to +\infty, y \to$ ____

5. Describe the transformation in the following equation: $f(x) = -3\left(\frac{5}{4}\right)^{x+4} - 2$

6. Describe the transformation in the following $f(x) = 2\left(\frac{1}{4}\right)^{x-3} + 1$

7. Write an equation that would be a decay by a factor of ½, a stretch of 3 and an asymptote of y = -2

8. What is the y-intercept of the equation you wrote in question 3?

Interpreting Exponential Functions

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from exponential functions (integer inputs only).
A.CED.2 Create exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 \;\; r/n)^{nt}$ has multiple variables.)
F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

NOTES: Exponential Growth and Decay Functions

$$y = ab^x$$

a:______________________________ b:______________________________

Growth Words	Decay Words

Example 1:
A population of mosquitoes doubles every day. There were originally 325 mosquitoes. Write a model for this situation. How many mosquitoes exist after 10 days?

Example 2:
In the science lab there are 150 bacteria in petri dish. The bacteria is dying at a rate of ½ every hour. How many bacteria will be alive after five hours?

Example 3:
Jen told 3 friends an interesting secret about someone they all know. They each told 3 people who also told 3 people. How many people will know this secret after 5 days?

Example 4:
A contagious virus affects triple the amount of people every week. If there are five sick people in Week 1, How many **weeks** will it take for 1215 people to be sick

NOTES: Exponential word problems
PERCENTS

$$y = P(1 \pm \%)^x$$

P:____

± ____

%:____

We call (1 + %) ____

We call (1 - %) ____

Converting % into decimal practice

In an equation, a percent must always be written in "_________________________". A percent is always a number out of ________. So if we want to convert a percent into a decimal we ____.

Convert the following percents into decimal form

14% =	**4% =**	**5% =**
1.34%	**0.6%=**	**50%**

Example 1:

It costs $400 to buy a football autographed by Falcon's quarterback Matt Ryan. It is expected increase in value 4% per year. How much do you expect it to be worth in 5 years?

Example 2:

A new iPad costs $650. It is expected to decrease in value 15% each year. How much do you expect it to be worth in 3 years?

Example 3: A house in my neighborhood is for sale for $150,000. It is expected to increase in value 1.2% per year. How much do you expect the house to be worth in 15 years? **Part b:** How much money did the house increase in value in over the 15 years?

Example 4: A brand new Jeep Wrangler cost $29,000. It is expected to decrease in value 12% per year. How much do you expect it to be worth in 7 years? **Part b:** How much money did the car lose in value over 7 years?

Saving Money: Future value of an investment

When we save money in an account, we typically deposit it with some account that will give us a percent interest. If we put a dollar into a piggy bank and never touch it again, in a year it will still only be one dollar. If we put a dollar into a savings account that is awarding us **interest** then that dollar could turn into $1.50 in a year. How much we gain is based on the interest and the **compounding** period. Compounding period refers to how often the interest is calculated on the current amount. Below is the formula used to find the future value of an investment based on interest and compounding periods.

$$FV = P\left(1 + \frac{r}{n}\right)^{nt}$$

What do the variables stand for?

FV	P	r	n	t

What are the different compounding periods?

Annual	Semi-annual	Quarterly	Monthly	Weekly	Daily

1. Jackson has $2,600 he is going to deposit into a savings account with a 4.25% interest compounded quarterly. How much will he have after 5 years? How much will he have after 10 years? How much will he have after 20 years?

5 years	
10 years	
20 years	

What if the same account compounded monthly? How much would he have after each of the following years?

	Compounded Quarterly (copy answer from #1)	Compounded Monthly	Amount increase?
5 years			
10 years			
20 years			

3. What can you determine about total money saved (future value) and compounded period?

4. Kami has $5,000 she is going to invest and has two different options

Savings account	Money Market account
• 3.15% interest rate • Compounded monthly	• 3.98% interest rate • Compounded semi-annually

Determine Kami's account balance after the following years

	Savings account	Money Market account
5 years		
10 years		
20 years		

5. Which has a greater impact on future value, compounding period or interest rate? Why?

7. Sometimes we like to work backwards when planning for a specific purpose. Like saving for a car, saving for college, saving for a house, or saving for retirement. Jada wants to start saving for retirement. A general rule of thumb is that you should have $1,000,000 saved by the time you retire. Let's assume that Jada is 20 years old and plans to retire at 67. She plans on opening up a savings account that has 1.2% interest compounded quarterly. Let's assume she does not make any additional deposits into her account (Even though this is unrealistic). How much money will she need to put away right now in order to have $1,000,000 when she retires?

8. Jesus is saving up to have a down payment on a new car when he graduates from college. He wants to have $3,500 saved. He plans on depositing a lump sum into a CD (Certificate of Deposit) account and withdrawing it after graduation. The CD account from Bank of America offers 0.15% interest compounded monthly. If we assume he does not make any deposits or withdraws for 4 years, how much will he need to deposit now in order to have the amount needed for a down payment? (and yes, you should be surprised by that number because the interest rate is so small)

1) Ryan is saving for his college tuition. He has $2,550 in a savings account that pays 6.25% annual interest. How much money will Ryan have in his account 6 years from now?

Formula:

2) Paul is saving money for college. He put $500 in a savings account. He plans to save $250 each month. How much money will Paul have in his account 3 months from now?

Formula:

3) In 2012 a used car was purchased for $12,329 this year. Each year the car's value decreases 8.5%. What will the car be worth in 2020?

Formula:

4) Misty gets paid 7.25 per hour working at Target. How much money will Misty make if she works 22.5 hours this week?

Formula:

5) Jeremiah owns a business. His first year he made $11,212, each of the following years his profit increased 12%. What will she make in 20 years?

Formula:

6) You want to clean your carpet. The company charges $20 to rent the cleaner plus $12 for each hour you have the cleaner. How much money will you pay if you rent the cleaner for 12 hours?

Formula:

7) Dianna is researching the reproductive rate of genetically modified vegetables to see if they will grow better than others. Right now she has planted 150 plants and they are doubling every year. How many plants can she expect to have after 10 years?

Formula:

8) Rob needs to call an electrician to fix some wiring. The electrician charges a service charge of $50 just to come out to the house and then $25 for each hour he is there. How much money will Rob spend if it takes the electrician 4 ½ hours to complete the job?

Formula:

9) A radioactive material decays at a rate of 40% per hour. You begin with 80 grams of radioactive material. How much will be remaining after 6 hours? How about after 24 hours?

Formula:

10) Globo Inc. is a company that makes money by getting clients to refer new customers to the company. It started with only 5 clients. If each client is required to get 3 new clients every month, how many clients will Globo Inc. have in a year?

Formula:

1. You deposit $1500 in an account that pays 5% interest compounded yearly. How much do you expect to have in 20 years?

2. The mice population is 25,000 and is decreasing by 20% each year. How many mice do you expect there to be in 4 years?

3. The number of mosquitoes at the beach increases at a rate of 6% each year. In 2000 there were 8,000 mosquitos. How many mosquitos do you expect in 2013? (hint: for x think of how many years it is between 2000 and 2013)

4. I bought a car for $25,000, but its value is depreciating at a rate of 10% per year. How much money can I expect the car to be worth 6 years from now?

5. I bought issue one of a comic that was signed by the artist for $65. I expect the price to increase by 30% each year. How much do you expect I will be able to sell it for in 15 years?

6. James bought a laptop for $999. The value depreciates by 18% each year. How much should he expect his laptop to be worth in 5 years?

7. The University of Georgia has increased the number of freshman it takes each year. This year they accepted 4,518 freshman. If the acceptance rate grows 1.3% each year, how many Freshmen do you expect them to take in 4 years?

8. Studies have shown that America is one of the most obese countries in the world. In a 2012 study, there were 105,000,000 obese people in the United States. If obesity rates are increasing 4.6% each year, how many people do you expect will be obese in America in 10 years? **PART B:** how many more people became obese in those 10 years?

1) A scientist is working with 750 grams of a radioactive material that has a half-life in hours. How much of the substance is left after four hours?

---

2) Ryan is saving for his college tuition. He has $2,550 in a savings account that pays 6.25% annual interest.

a) Write an exponential equation describing this situation. _______________

b) How much money will Ryan have in his account 6 years from now?

---

3) A used car was purchased for $12,329 this year. Each year the car's value decreases 5.5%.

a) Write an exponential equation describing this situation. _______________

b) What will the car be worth in 2022?

---

4) The number of mosquitoes at the beach has tripled every year since 1999. In 1999, there were 2,500 mosquitoes.

a) Write an exponential model for this situation. _______________

b) How many mosquitoes would you predict were at the beach in 2005?

---

5) Jeremiah owns a business. His first year he made $11,212, each of the following years his profit increased 12%.

a) Write an exponential equation describing the situation. _______________

b) What will he make in 20 years?

c) How many years will it take to make over $200,000?

---

6) Dianna just bought a home. She paid $240,000. She is able to pay 20% of the loan off each year.

a) Write an exponential equation describing the situation. _______________

b) What will she owe in 10 years?

---

7) A radioactive material decays at a rate of 40% per hour.

a) If we start with 80 grams of the substance, can you find a formula that models this rate of decay? _______________

b) How much will be remaining at the end of 6 hours?

Geometric Sequences

F.BF.2 Write geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect geometric sequences to exponential functions

F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (Generally, the scope of high school math defines this subset as the set of natural numbers 1,2,3,4...) By graphing or calculating terms, students should be able to show how the recursive sequence $a_1 = 7$, $a_n = a_{n-1}\ 2$; the sequence $s_n = 2(n - 1)\ 7$; and the function $f(x) = 2x\ 5$ (when x is a natural number) all define the same sequence.

NOTES: Linear Sequences (Arithmetic Sequences)

Arithmetic Sequence Formulas

Recursive:	$a_n = a_{n-1} + d$
Explicit:	$a_n = a_1 + (n - 1)d$

This is STRAIGHT from the EOC formula sheet

Vocabulary:

Sequence means :____

Arithmetic means :____

____

Recursive means:____

____

Explicit means: ____

____

What are the variables?

USED IN BOTH:

a_n: ____

d: ____

USED IN RECURSIVE

a_{n-1}: ____

USED IN EXPLICIT

n: ____

a_1: ____

3, 9, 15, 21, 27…..

a_1= ______ d = ______
What is the 4th term?
9 represents which term?

Examples

1) Write a recursive rule for the following sequence 2, 6, 10, 14, 18…..	2) Write a recursive rule for the following sequence 3, 7, 11, 15, 19…..
3) Write an explicit rule for the following sequence 11, 20, 29, 38, 47 ….	4) Write an explicit rule for the following sequence 3, 7, 11, 15, 19…..
5) What is the twentieth term of the sequence whose nth term is $a_n = -3n + 14$	6) What is the sixtieth term of the sequence whose nth term is $a_n = -8n - 1$
7) Find the 36th term of the sequence 26, 24, 22, 20, 18, 16…..	8) Find the 28th term of the sequence 13, 27, 41, 55, 69…

NOTES: Exponential Sequences (Geometric)

Geometric Sequence Formulas

Recursive: $a_n = r(a_{n-1})$

Explicit: $a_n = a_1 \cdot r^{n-1}$

This is STRAIGHT from the EOC formula sheet

What are the variables?

USED IN BOTH:

a_n: ______________________________

r : ______________________________

USED IN RECURSIVE

a_{n-1}: ______________________________

USED IN EXPLICIT

n: ______________________________

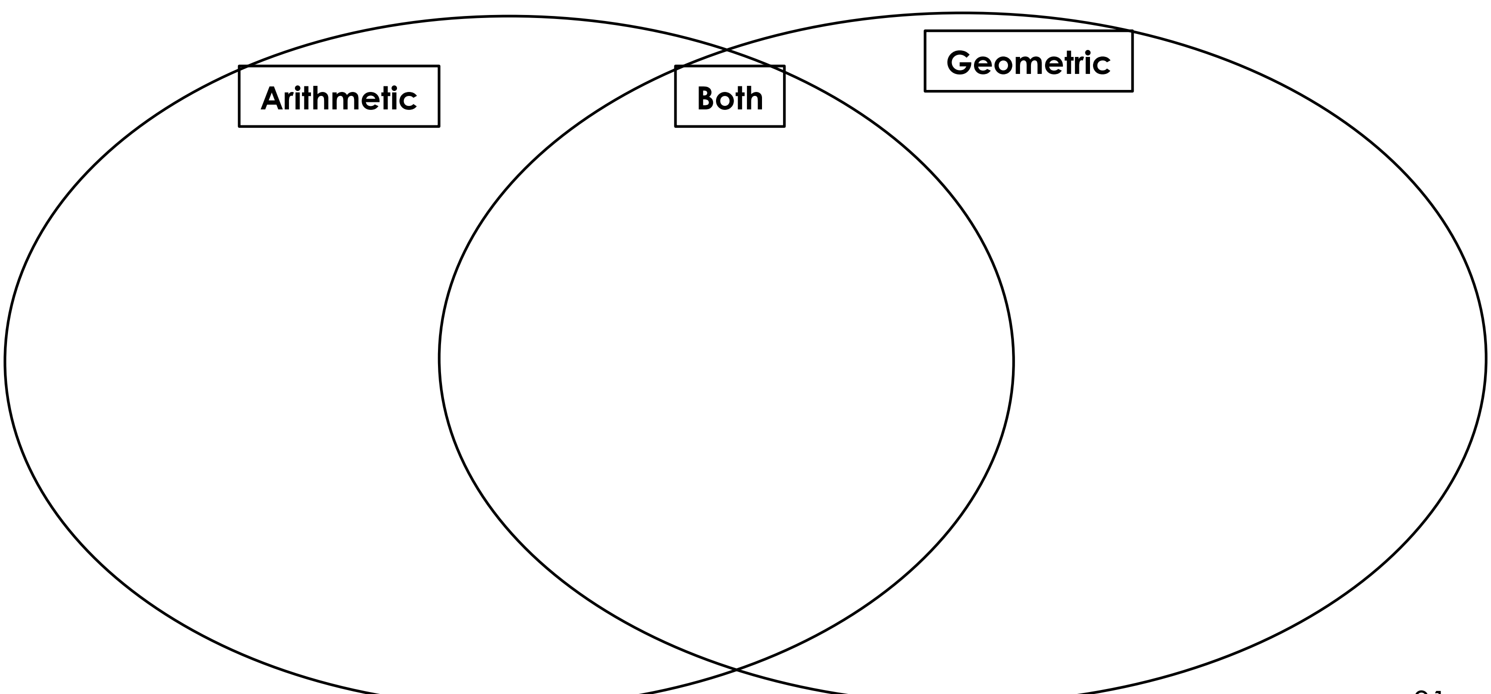
a_1: ______________________________

4, 8, 16, 32, ….

a_1= _______ r = _______

What is the 4th term?

8 represents which term?



Examples

1) Write a recursive rule for the following sequence 5, 50, 500, 5000…	2) Write a recursive rule for the following sequence -4, -24, -144, -864…
3) Write an explicit rule for the following sequence 5, 50, 500, 5000…	4) Write an explicit rule for the following sequence -4, -24, -144, -864…
5) What is the twentieth term of the sequence whose nth term is $a_n = -3(2)^{n-1}$	6) What is the sixteenth term of the sequence whose nth term is $a_n = 2(-3)^{n-1}$
7) Find the 9th term of the sequence 2, 6, 18, 54….	8) Find the 15th term of the sequence -6, -18, -54, -162…

Arithmetic and Geometric Sequences Practice - Explicit and Recursive

1) **1, 4, 7, 10... Arithmetic or Geometric?**________________________________

b) Find the next term in sequence: ____________________________________

c) Write the Explicit Rule: ____

d) Find the 22^{nd} term: ____

e) Write the Recursive Rule:____

2) **4, -12, 36, -108... Arithmetic or Geometric?**________________________________

a) What is the common ratio/common difference?:______________________

b) Find the next term in sequence: ____________________________________

c) Write the Explicit Rule: ____

d) Find the 9^{th} term: ____

e) Write the Recursive Rule:____

3) **3, 203, 403... Arithmetic or Geometric?**________________________________

a) What is the common ratio/common difference?:______________________

b) Find the next term in sequence: ____________________________________

c) Write the Explicit Rule: ____

d) Find the 20^{th} term: ____

e) Write the Recursive Rule:____

4) 2, 4, 8, 16... Arithmetic or Geometric?________________________________

a) What is the common ratio/common difference?:____________________

b) Find the next term in sequence: __________________________________

c) Write the Explicit Rule: ____

d) Find the 15th term: ____

e) Write the Recursive Rule:_______________________________________

5) -2, 5 12, 19... Arithmetic or Geometric?________________________________

a) What is the common ratio/common difference?:____________________

b) Find the next term in sequence: __________________________________

c) Write the Explicit Rule: ____

d) Find the 13th term: ____

e) Write the Recursive Rule:_______________________________________

6) -2, 6, -18, 54... Arithmetic or Geometric?________________________________

a) What is the common ratio/common difference?:____________________

b) Find the next term in sequence: __________________________________

c) Write the Explicit Rule: ____

d) Find the 15th term: ____

e) Write the Recursive Rule:_______________________________________

7) 12, -11, -34, -57 Arithmetic or Geometric?______________________________

a) What is the common ratio/common difference?:____________________

b) Find the next term in sequence: __________________________________

c) Write the Explicit Rule: ____

d) Find the 50th term: ____

e) Write the Recursive Rule:_______________________________________

Assignment

Date________________ Period____

Find the common difference, the 52nd term, the explicit formula, and the recursive formula.

1) $-28, -20, -12, -4, \ldots$

2) $29, -71, -171, -271, \ldots$

Find the common ratio, the 8th term, the explicit formula, and the recursive formula.

3) $4, -16, 64, -256, \ldots$

4) $-2, 6, -18, 54, \ldots$

Find the common difference, the 52nd term, the explicit formula, and the recursive formula.

5) $-8, -17, -26, -35, \ldots$

6) $-33, -25, -17, -9, \ldots$

Determine if the sequence is geometric. If it is, find the common ratio, the 8th term, the explicit formula, and the recursive formula.

7) $3, 6, 12, 24, \ldots$

8) $2, 4, 8, 16, \ldots$

Find the recursive formula.

9) $31, 25, 19, 13, \ldots$

A) $a_n = a_{n-1} - 6$, $a_1 = 13$

B) $a_n = a_{n-1} - 6$, $a_1 = 19$

C) $a_n = a_{n-1} - 6$, $a_1 = 31$

D) $a_n = a_{n-1} - 6$, $a_1 = 25$

10) $19, 11, 3, -5, \ldots$

A) $a_n = a_{n-1} + 8$, $a_1 = 19$

B) $a_n = a_{n-1} + 10$, $a_1 = 29$

C) $a_n = a_{n-1} + 10$, $a_1 = 19$

D) $a_n = a_{n-1} - 8$, $a_1 = 19$

Find the explicit formula.

11) $-1,\ 5,\ -25,\ 125,\ \ldots$

A) $a_n = -5^{n-1}$

B) $a_n = 5^{n-1}$

C) $a_n = -2 \cdot 5^{n-1}$

D) $a_n = -(-5)^{n-1}$

12) $2,\ -12,\ 72,\ -432,\ \ldots$

A) $a_n = 2 \cdot (-6)^{n-1}$

B) $a_n = \frac{2}{5} \cdot 5^{n-1}$

C) $a_n = 2 \cdot 5^{n-1}$

D) $a_n = 5^{n-1}$

Find the term named in the problem.

13) $-13,\ -33,\ -53,\ -73,\ \ldots$
Find a_{32}

A) $a_{32} = -634$ B) $a_{32} = -674$

C) $a_{32} = -633$ D) $a_{32} = -654$

14) $-37,\ 163,\ 363,\ 563,\ \ldots$
Find a_{31}

A) $a_{31} = 5933$ B) $a_{31} = 6023$

C) $a_{31} = 5963$ D) $a_{31} = 5993$

Find the 8th term.

15) $-2,\ -12,\ -72,\ -432,\ \ldots$

A) $a_8 = -\frac{78125}{3}$

B) $a_8 = -93312$

C) $a_8 = -559872$

D) $a_8 = 729$

16) $-1,\ 4,\ -16,\ 64,\ \ldots$

A) $a_8 = 729$ B) $a_8 = 6561$

C) $a_8 = 16384$ D) $a_8 = 2187$

Find the recursive formula.

1) 23, 3, −17, −37, ...

A) $a_n = a_{n-1} - 20$
$a_1 = 24$

B) $a_n = a_{n-1} - 20$
$a_1 = -16$

C) $a_n = a_{n-1} - 20$
$a_1 = 4$

D) $a_n = a_{n-1} - 20$
$a_1 = 23$

2) 33, −67, −167, −267, ...

A) $a_n = a_{n-1} - 99$
$a_1 = 33$

B) $a_n = a_{n-1} - 98$
$a_1 = 33$

C) $a_n = a_{n-1} - 100$
$a_1 = -67$

D) $a_n = a_{n-1} - 100$
$a_1 = 33$

Find the explicit formula.

3) −4, −12, −36, −108, ...

A) $a_n = 4 \cdot 5^{n-1}$

B) $a_n = 4 \cdot \left(\frac{1}{5}\right)^{n-1}$

C) $a_n = -4 \cdot 3^{n-1}$

D) $a_n = 4 \cdot (-3)^{n-1}$

4) 2, 10, 50, 250, ...

A) $a_n = 8 \cdot 5^{n-1}$

B) $a_n = 9 \cdot 5^{n-1}$

C) $a_n = 2 \cdot 5^{n-1}$

D) $a_n = 10 \cdot 5^{n-1}$

Find the term named in the problem.

5) −8, −208, −408, −608, ...
Find a_{26}

A) $a_{26} = -4958$ B) $a_{26} = -5008$
C) $a_{26} = -5354$ D) $a_{26} = -5156$

6) 25, 16, 7, −2, ...
Find a_{40}

A) $a_{40} = -344$ B) $a_{40} = -335$
C) $a_{40} = -326$ D) $a_{40} = -353$

Find the 8th term.

7) −4, −16, −64, −256, ...

A) $a_8 = -390625$

B) $a_8 = -65536$

C) $a_8 = -468750$

D) $a_8 = -312500$

8) −2, 12, −72, 432, ...

A) $a_8 = -\frac{1}{64}$ B) $a_8 = -256$

C) $a_8 = 559872$ D) $a_8 = \frac{1}{32}$

Find the common difference, the 52nd term, the explicit formula, and the recursive formula.

9) 26, 22, 18, 14, ...

10) 6, 0, −6, −12, ...

Find the common ratio, the 8th term, the explicit formula, and the recursive formula.

11) 2, 6, 18, 54, ...

12) 2, −8, 32, −128, ...

Find the common difference, the 52nd term, the explicit formula, and the recursive formula.

13) 0, 5, 10, 15, ...

14) 19, 14, 9, 4, ...

Determine if the sequence is geometric. If it is, find the common ratio, the 8th term, the explicit formula, and the recursive formula.

15) −2, 8, −32, 128, ...

16) 3, 18, 108, 648, ...