Honors Algebra 1

Unit 5:
Comparing and Contrasting Functions

Name:_____

Fall 2019
Dr. Oldham

Comparing linear, quadratic, and exponential functions and graphs

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. (This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals).
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum..

F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and $f(x)$ is the output (an element of the range). Graphically, the graph is $y = f(x)$.

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

NAME OF FUNCTION	GENERAL SHAPE OF GRAPH	SKETCH
LINEAR		
QUADRATIC		
EXPONENTIAL		

1. Complete the following tables and answer the questions to the right.

(a)

X	y = 2x	1st Diff
-3		
-2		
-1		
0		
1		
2		
3		

This function is.

☐ linear ☐ quadratic ☐ exponential

Describe two ways you were able to determine what this function is:

(b)

x	$y = x^2$	1st Diff	2nd Diff
-3			
-2			
-1			
0			
1			
2			
3			

This function is.

☐ linear ☐ quadratic ☐ exponential

How do you know?

(c)

x	$y = 2^x$	1st Diff	2nd Diff
-3			
-2			
-1			
0			
1			
2			
3			

What do you notice about the differences in this function?

_____

By what number is the first difference multiplied by to get the next term in the sequence of y-values?

_____

How does this value connect to the function?

_____

This function is. □linear □ quadratic □ exponential

What methods can you use to verify the type of function selected?

2. Use differences to identify the type of function represented by the table of values. Then label which type of function each table of values models.

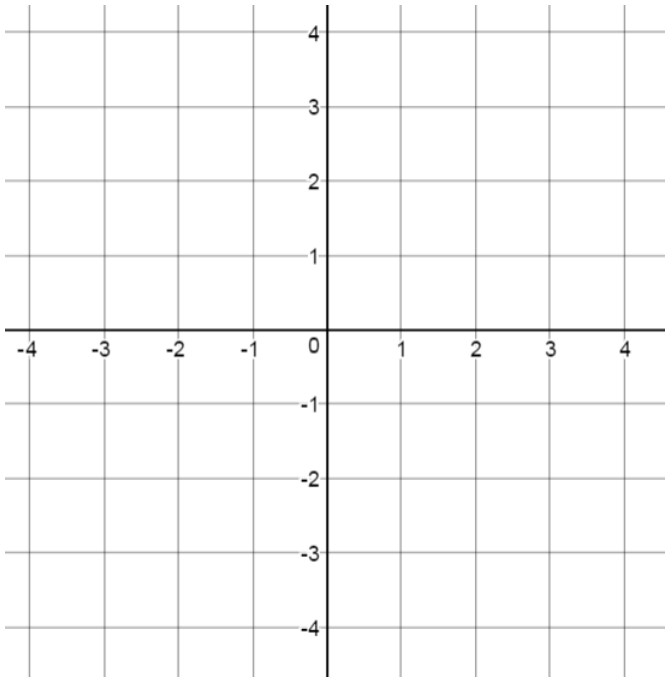
x	y		x	y		x	y		x	y
-4	5		-5	32		-2	8		0.5	0.9
-3	8		-4	16		-1	4		0.75	1.1
-2	13		-3	8		0	2		1	1.3
-1	20		-2	4		1	1		1.25	1.5
0	29		-1	2		2	.5		1.5	1.7
1	40		0	1		3	.25		1.75	1.9

Function: _____ Function: _____ Function: _____ Function: _____

Identify the following equations as linear, quadratic or exponential.

1. $y = 10\left(\frac{1}{3}\right)^x$ □linear □ quadratic □ exponential	**2.** $y = 5 + 7(x)$ □linear □ quadratic □ exponential
3. $y = (x+3)^2 - 4$ □linear □ quadratic □ exponential	**4.** $y = -2(x) + 5$ □linear □ quadratic □ exponential
5. $y = -\frac{1}{2}(3)^x$ □linear □ quadratic □ exponential	**6.** $y = \frac{1}{3}(x)^2 - 4$ □linear □ quadratic □ exponential

Graph the functions $y = 2x$, $y = x^2$ and $y = 2^x$ on the same grid for**. Label your graphs.**



Looking at the graphs above:

a) Which function equation shows a constant rate of change in its y values? How is this displayed on your graph?

b) For $x < 4$ how are the rates of change different for each graph?

Practice Problems

Identify the following equations as linear, quadratic or exponential.

1. $y = 4^x + 6$

2. $y = -\frac{3}{2}x - 3$

3. $y = x^2 - 5x + 6$

4. $y = -2(4)^x$

5. $y = 3x + 3$

6. $f(x) = (x-2)^2 + 7$

Function Identification

Determine if the following tables are quadratic, exponential, linear, or neither.

x	-3	-2	-1	0	1	2	3
y	14	10	6	2	-2	-6	-10

x	-3	-2	-1	0	1	2	3
y	21	12	5	0	-3	-4	-3

x	-3	-2	-1	0	1	2	3
y	-16	-13	-10	-7	-4	-1	2

x	-3	-2	-1	0	1	2	3
y	-14	-9	-4	1	6	11	16

x	-3	-2	-1	0	1	2	3
y	-18	-6	-2	0	2	6	18

x	-3	-2	-1	0	1	2	3
y	4	8	16	32	64	128	256

x	-3	-2	-1	0	1	2	3
y	9/8	9/4	9/2	9	18	36	72

x	-3	-2	-1	0	1	2	3
y	30	20	12	6	2	0	0

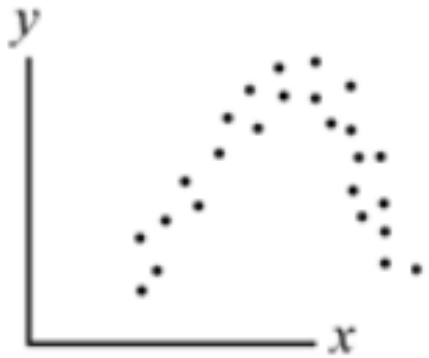
x	-3	-2	-1	0	1	2	3
y	1/27	1/9	1/3	1	3	9	27

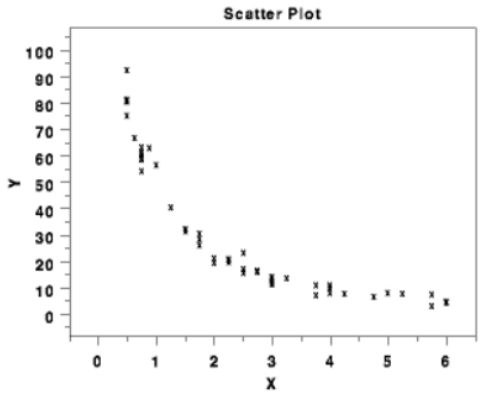
x	-3	-2	-1	0	1	2	3
y	1/9	1/3	1	3	9	27	81

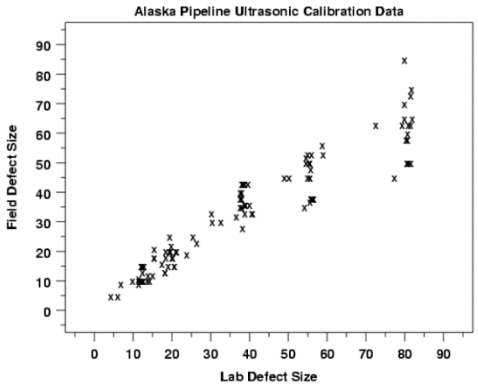
x	-3	-2	-1	0	1	2	3
y	11	9	7	5	3	1	-1

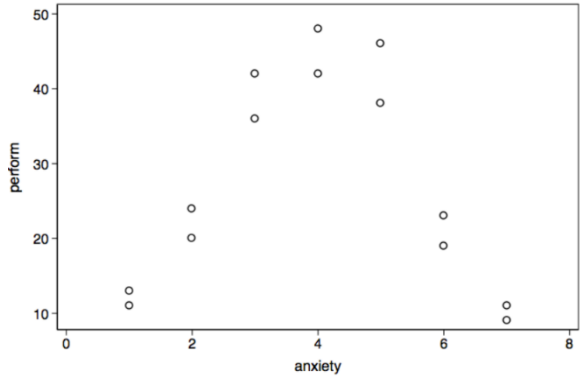
x	-3	-2	-1	0	1	2	3
y	0	5	8	9	8	5	0

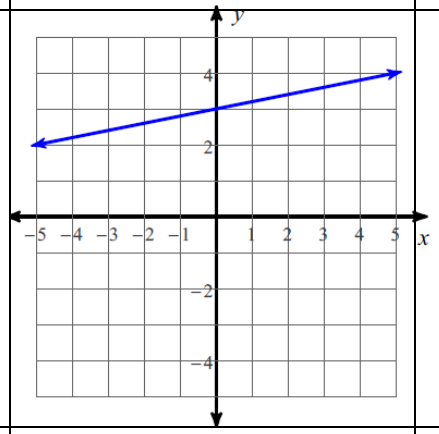
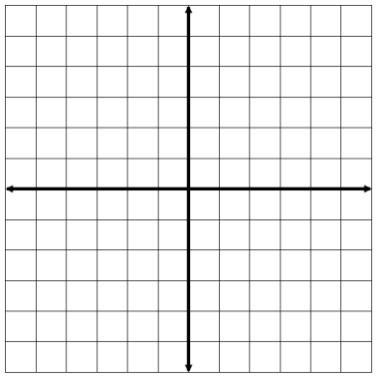
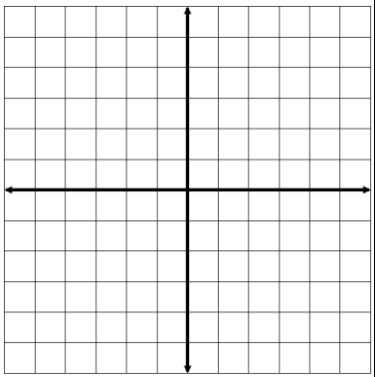
$2x^2+4x-3$	$4x-5$	$3^{x-2}+4$	$2(x-1)^2+1$	$3x-4y=12$

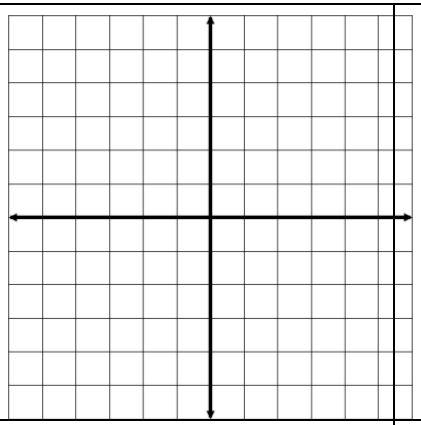
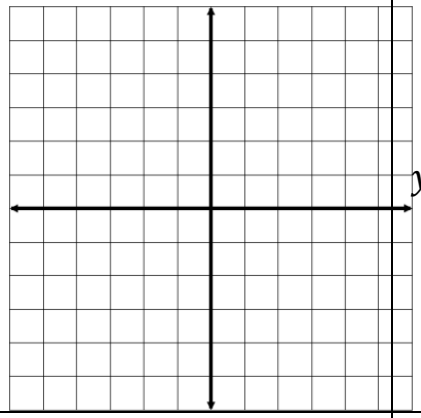
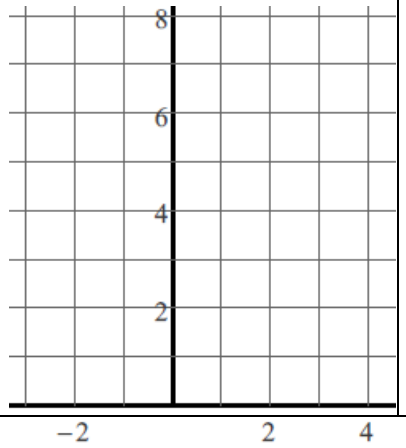


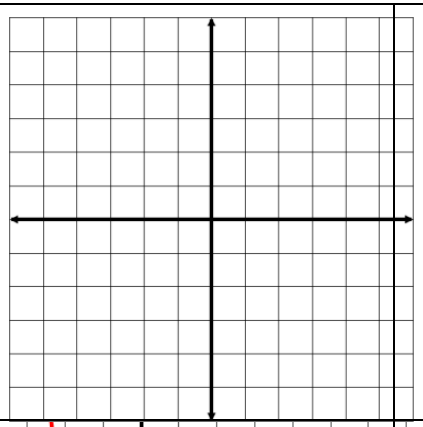
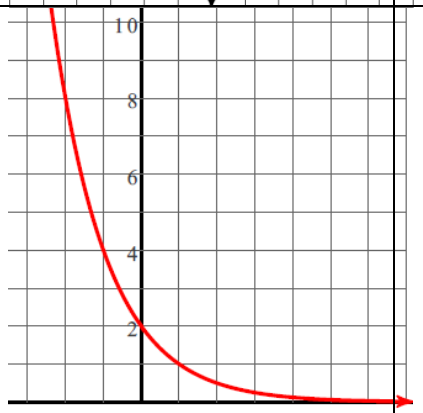
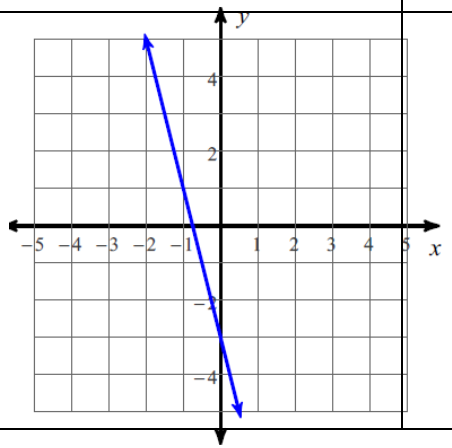


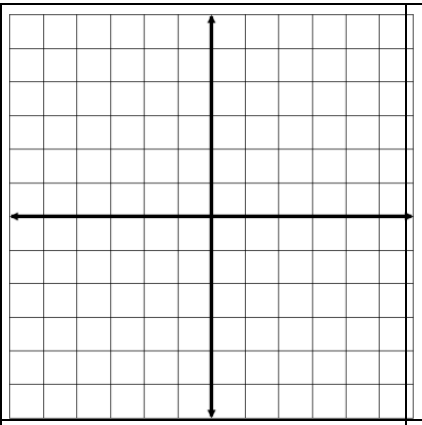
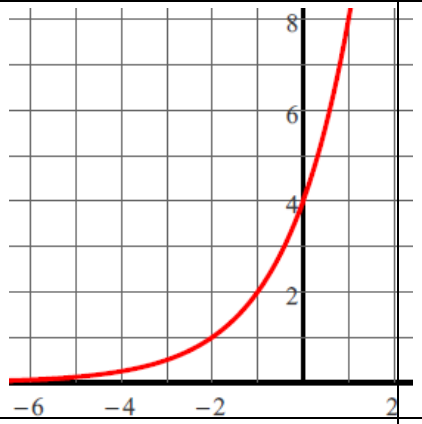
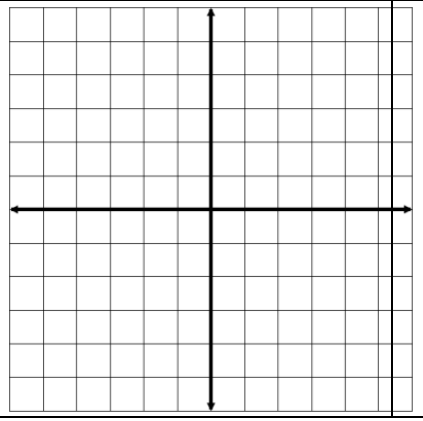




FUNCTION	TABLE	GRAPH	EQUATION	AROC
	<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td></tr></table>			[-2, 0]
	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>y</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr></table>			[0, 3]
	<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td>12</td><td>6</td><td>3</td><td>1.5</td><td>.75</td></tr></table>			[-2, 0]

	<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td></tr></table>		$f(x) = \frac{2}{5}x + 1$	[-1, 2]
	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td></tr></table>		$y = x^2 - 4x + 5$	[0, 2]
	<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td>1</td><td>2</td><td>4</td><td>8</td><td>16</td></tr></table>			[1, 4]

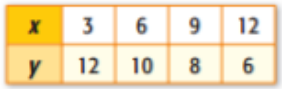
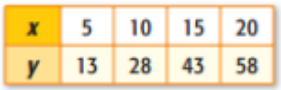
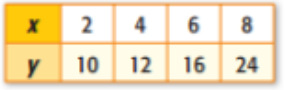
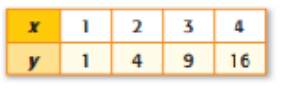
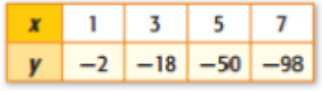
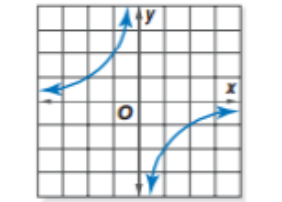
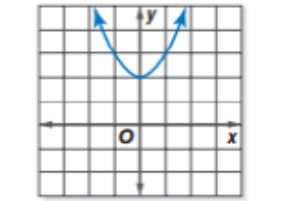
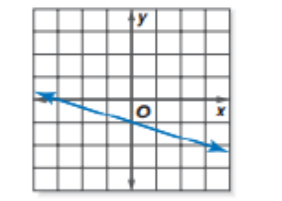
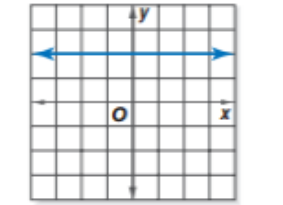
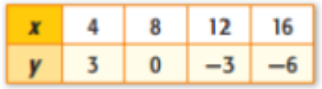
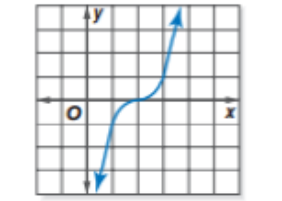
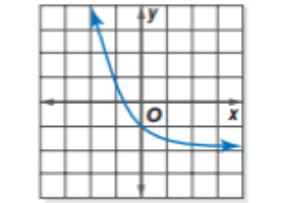
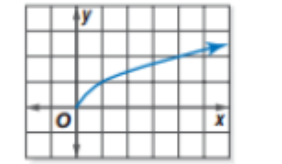
	<table><tr><td>x</td><td>-4</td><td>-3</td><td>-2</td><td>-1</td><td>0</td></tr><tr><td>y</td><td>4</td><td>1</td><td>-2</td><td>-5</td><td>-8</td></tr></table>			[-4, 0]
	<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td></tr></table>			[0, 2]
	<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td></tr></table>			[-1, 5]

	<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td>-6</td><td>-5</td><td>-4</td><td>-3</td><td>-2</td></tr></table>			[-2, 2]
	<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td></tr></table>			[-1, 2]
	<table><tr><td>x</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td></tr></table>		$x^2 + 2x - 5$	[-3, 0]

Linear, Exponential, Quadratic or Neither

KEY
= LINEAR
= EXPONENTIAL
= QUADRATIC
= NEITHER

Determine a color for each type of function. Color the box the appropriate color based on what type of function it is.

$y = 7x + 4$	$4x - y = 13$	$y = x^3 + 2x^2 - 12$	$y = x^2 - 1$	$(x - 2)(x - 3) = 12$
$3x - \frac{4}{3}y = 15$	$y = x(x + 9)$	$y = -3(x - 2)^2 + 8$	$y = 6x^5$	$y = \frac{13}{x}$
$-x^2 - 2x + 3 = 0$	$x^4 - 10 = 0$	$xy = 17$	$f(x) = 2^{x-3}$	$y = \frac{3x}{2}$
				
				
			$y = \frac{3}{4}(x + 12) - 2$	$y - 1 = -2(x - 5)$
$y = 14$	$9 = xy$	$\frac{x}{y} = 18$	$\{(1,2), (3,4), (5,6), (7,8), (9,10\}$	$\{(1,2), (1,3), (1,4), (1,5), (1,6\}$
A golf ball was hit at the driving range.	The carnival charged an entry fee to get in the park and also a fee per ride ticket.	The population of beetles doubled every week.	Mr. Green weighed decided he wanted to lose four pounds a week.	A basketball free throw was shot to win the game.
$y = (x - 3)(x + 6)$	$x^2 = 16$	The stock market loses a third of its value every ten years.	The flight path of an Angry Bird.	$\{(1, 1), (2, 4), (3, 9), (4, 2), (5, 1)\}$

Spreading a rumor

Keisha and Jenny have very bad habits or spreading rumors. They love to gossip.

- When Keisha spreads a rumor she tells 10 people of the first day and then tells 5 people each day after that

- When Jenny spreads a rumor she tells 3 people the first day and then doubles the amount of people she tells each day after that.

Keisha

Write a model for this situation:

Fill out the chart for the first 5 days:

Days	# of people who know the rumor
0	
1	
2	
3	
4	
5	

Jenny

Write a model for this situation:

Fill out the chart for the first 5 days:

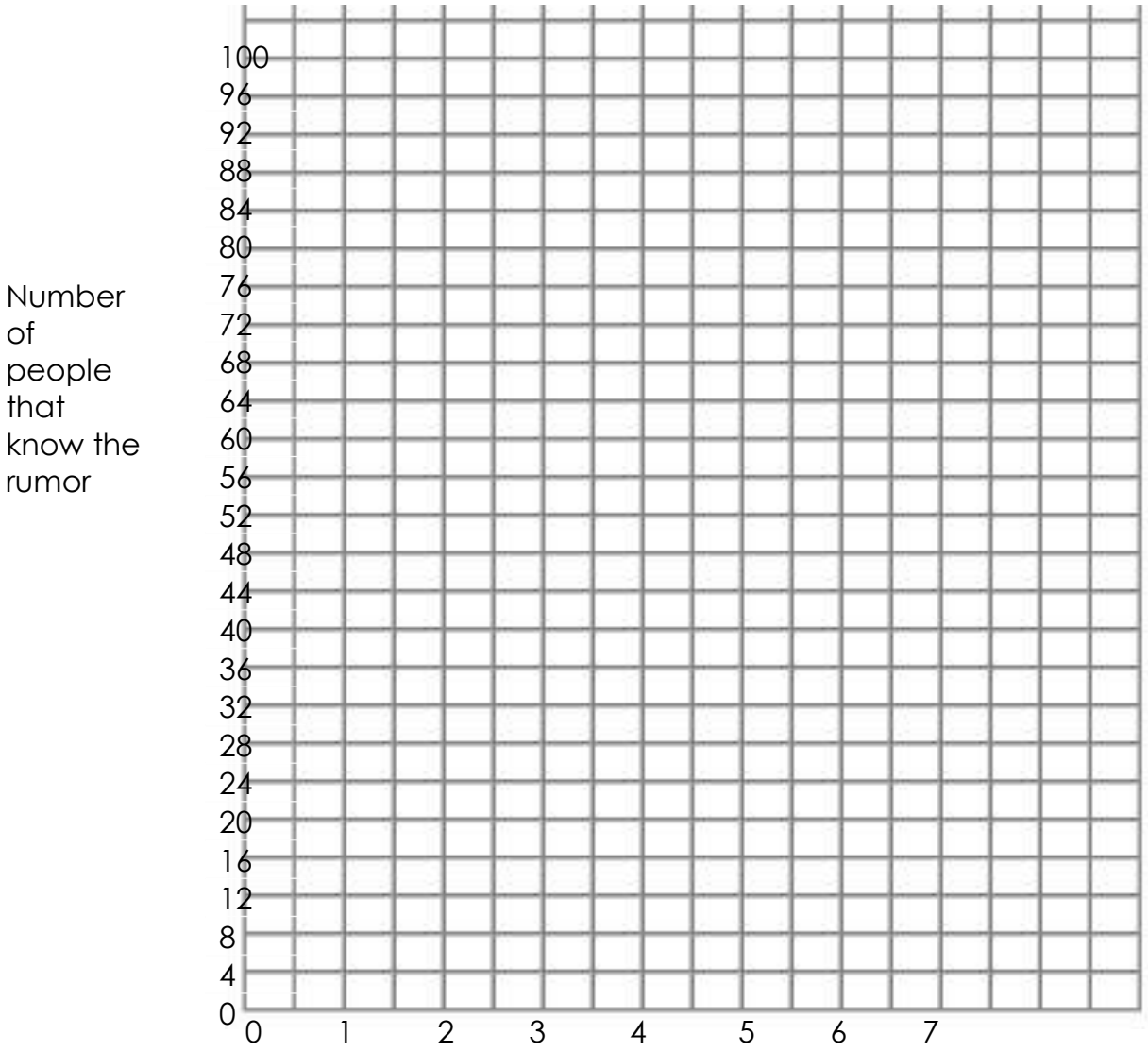
Days	# of people who know the rumor
0	
1	
2	
3	
4	
5	

1. Who has told more people the rumor initially?
   Keisha Jenny

2. Who has told more people the rumor on day 2?
   Keisha Jenny

3. Who has told more people the rumor on day 3?
   Keisha Jenny

4. Who has told more people the rumor on day 4?
   Keisha Jenny



5. How many people will Keisha have told the rumor to in 10 days? How many people will Jenny have told the rumor to in 10 days?

6. What kind of graph is Keisha's model? What kind of graph is Jenny's model?

7. If these girls were spreading a rumor about you, which girl would you rather be spreading the rumor? Why?(remember the secret is about you so you probably don't want that many people to know about it)

Paying off a loan

Luke is paying off a loan on his brother who loaned him money to buy a new mountain bike. The bike cost him $400. Luke's brother is giving him two options to pay him back

- Luke can pay back 7% of the cost every month

- Luke can pay back $25 each month

Option 1

Write a model for this situation:

Fill out the chart for the first 9 months:

Month	Amount of loan remaining
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Option 2

Write a model for this situation:

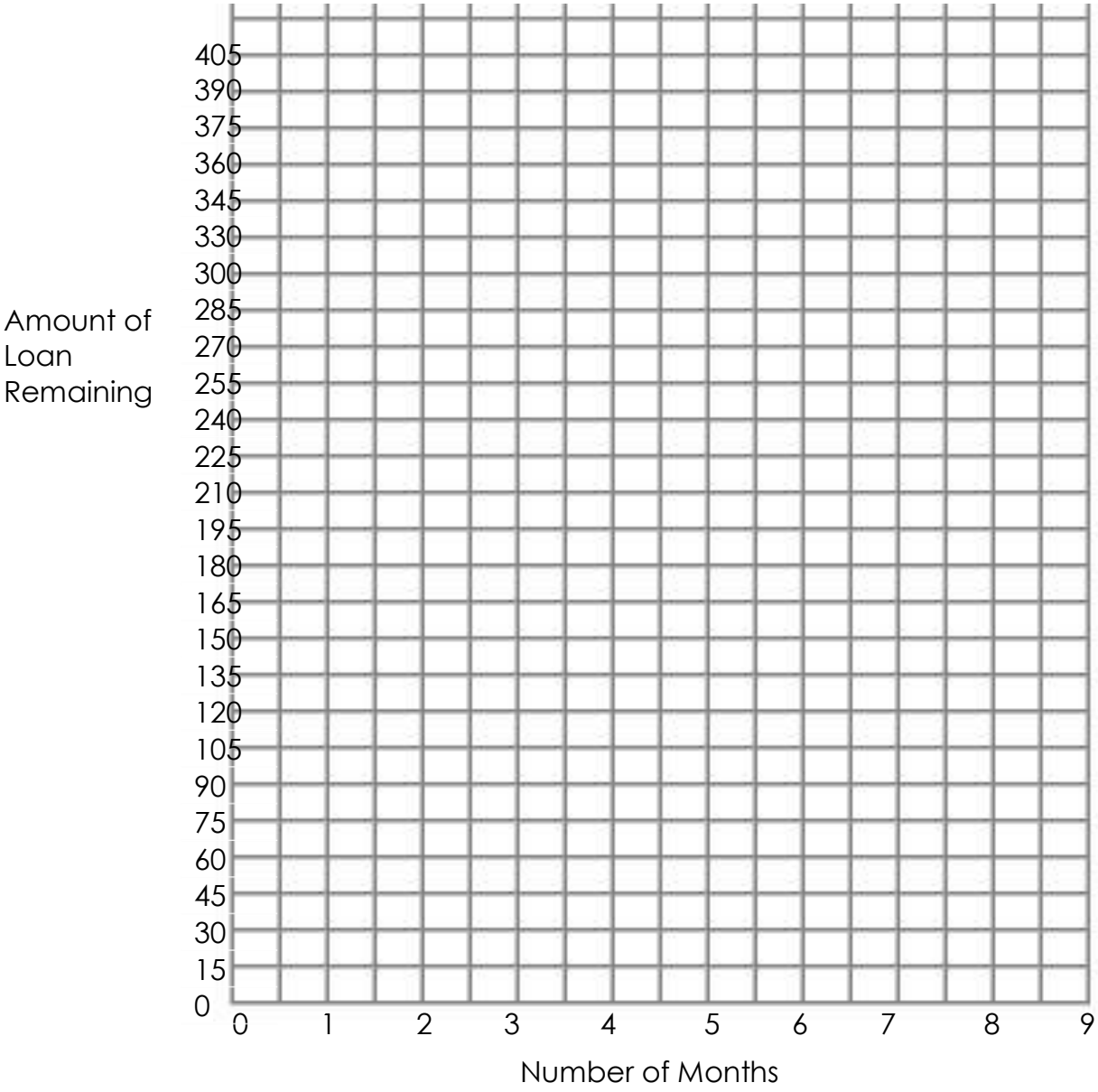
Fill out the chart for the first 9 months:

Month	Amount of loan remaining
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

1. By month 3 which option has the least amount of loan remaining?
   Option 1 Option 2

2. By month 9 which option has the least amount of loan remaining?

   Option 1 Option 2



3. How long will it take to pay off the loan in option 2?

4. How much will Luke still owe in 16 months with Option 1?

5. What kind of function is option 1? What kind of function is option 2?

6. Which option would you suggest that Luke take from his brother? Why?

Linear and Exponential Sequences

1. Jack and Jane are siblings. The made a new years resolution to save money. The both opened up a savings account on January 1st. Jack deposited $500 in his savings account that is compounded monthly with an interest rate of 7.8%. On the same day Jane deposited $450 in a savings account that is compounded monthly with an interest rate of 15.4%.
    a. How much money will Jack have after 3 years?

    b. How much money will Jill have after 3 years?

    c. Who has more money?

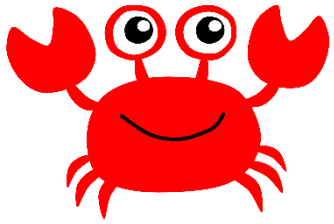
    d. By how much?

2. Harrison collects arrowheads. He is trying to get more than his friend Greg. Right now he has five and he is hoping to double the amount he has each week.
    a. Write a sequence that shows how many arrowheads he will have for the first 4 weeks.

    b. Greg has 1,000 arrowheads. How many weeks will it take for Harrison to have more arrowheads than Greg?

3. Polly is saving up to buy a computer for when she goes to college. She wants to have $800 saved when she graduates in 4 years. If she has a savings account that pays 8.2% interest each year, how much money should she put down now in order to have enough saved?

4. On a remote island there are 300 crabs.
   a. Write an equation that represents the population if it is decreasing by 2% each year.



   b. Write an equation that represents the population if it is increasing by 2% each year.

5. For each of the following determine the PERCENT that each sequence is changing by: 5%, 10%, 15%, 20%, 25%, 50%
   a. 60, 48, 38.4, 30.72…
   b. 20, 22, 24.2, 26.62…
   c. 10, 7.5, 5.63, 4.22….
   d. 40, 20, 10, 5, 2.5…
   e. 8, 8.4, 8.82, 9.26…
   f. 10, 11.5, 13.23, 15.21…

6. **Write an equation that represents the formula for the nth term**
   NOTE: sometimes the question is asked this way when it is just asking you to write the equation. For example $y = 4x - 1$ is the same thing as $a_n = 4n - 1$. So if it asks to write for the nth term you just need to change your variables from y to a_n and x to n
   a. 6, 14, 22, 30, 38…
   b. 1, 10, 100, 1000…
   c. 5, 40, 320, 2560…
   d. -1, 0, 1, 3, 5…
   e. 4, 1, -2, -5, -8…
   f. 2, -8, 32, -128

Height vs. Age charts are an important way to keep track of a child's progress.

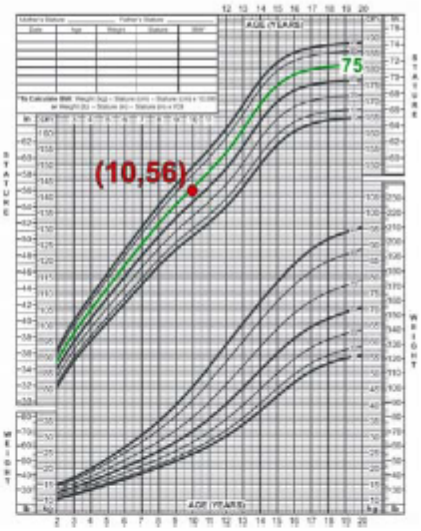
Read the information on this page about Height vs. Age charts. Then, use the Height vs. Age charts for girls and boys to examine rates of change and answer the questions on the following page.

Height vs. Age Charts

Height vs. Age charts are used to keep track of a child's progress. They were developed by the Centers for Disease Control and Prevention (CDC), using data from thousands of children.

Based on height, these charts indicate the percentile at which a child falls compared to other children of the same age. Each curved line on the chart indicates a specific percentile. To use them, locate the point on the chart corresponding to a child's age along the horizontal axis and the child's height along the vertical axis. Then, identify the curve closest to that point, which indicates the percentile.

For instance, Monique is 10 years old and 56 inches tall. On the girls' Height vs. Age chart, the point (10 years, 56 inches) lies just below the curve for the 75th percentile. This means that Monique is taller than approximately 75% of all 10-year-old girls, but she is not as tall as the other 25% of 10-year-old girls.



Four separate charts are published by the National Center for Health Statistics (NCHS), a division of the CDC:

- Boys, birth to 36 months
- Boys, 3 to 18 years of age
- Girls, birth to 36 months
- Girls, 3 to 18 years of age

Parents often become fixated with their child's percentile, especially if their children fall below the 50th percentile. However, parents have nothing to worry about, as long as the child is consistently within the normal range. (The "normal range" is usually defined as a height between the 10th and 90th percentiles.) Physicians will become concerned, however, if a child repeatedly falls below the lower limit. A physician will also have concern if the percentile continues to decrease. For instance, a physician may worry about a child who measured at the 90th percentile at age two, but then fell to the 50th percentile by age three, and then fell to the 10th percentile by age four. From a health standpoint, it is important for children to maintain roughly the same percentile as they get older.

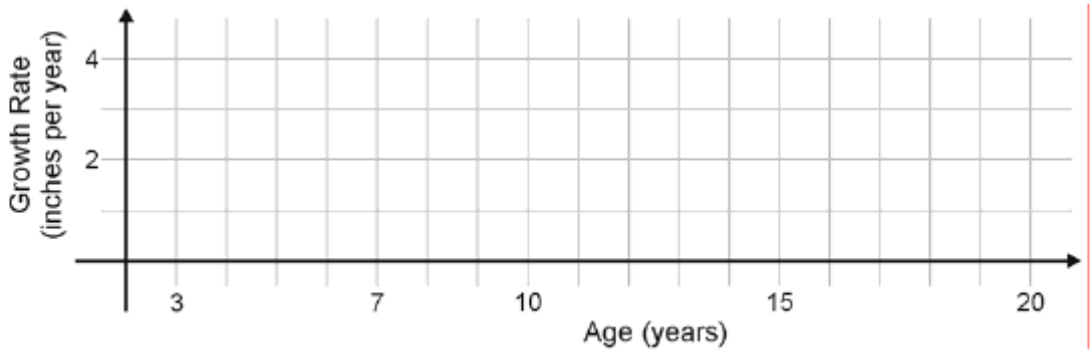
1. Consider a girl at the 50th percentile. Estimate the rate of change in her height (inches per year) at the following ages.

AGE	3	5	7	9	11	13	15	17
RATE OF CHANGE IN HEIGHT (INCHES PER YEAR)								

2. For a girl at the 50th percentile, at what age is her rate of change in height the greatest? That is, when is she growing the fastest? Estimate her rate of change in height (inches per year) at this age.

3. As a girl at the 50th percentile gets close to 20 years of age, explain what happens to her rate of change in height. Use the graph of Height vs. Age to justify your answer.

4. Use your results from Question 1 to sketch a graph representing the approximate **rate of change in height** vs. the age for a girl at the 50th percentile from age 3 to 20. What happens to a girl's rate of change in height over time? Explain.



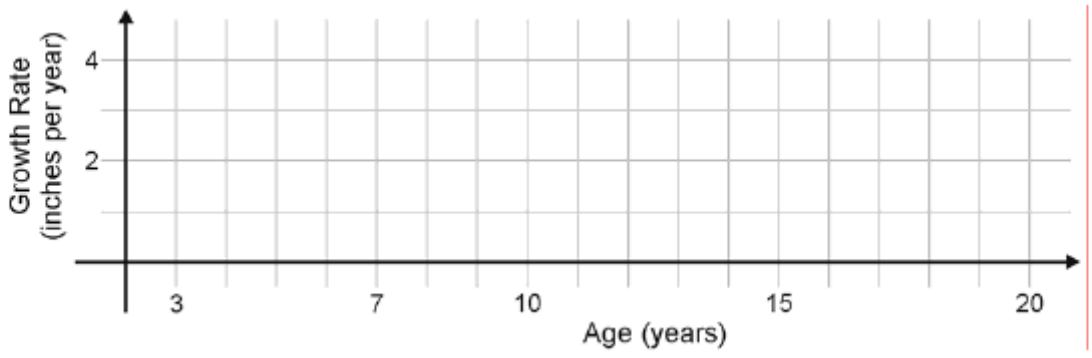
5. Consider a boy at the 50th percentile. Estimate the rate of change in his height (inches per year) at the following ages.

AGE	3	5	7	9	11	13	15	17
RATE OF CHANGE IN HEIGHT (INCHES PER YEAR)								

6. For a boy at the 50th percentile, at what age is his rate of change in height the greatest? That is, when is he growing the fastest? Estimate his rate of change in height (inches per year) at this age.

7. As a boy at the 50th percentile gets close to 20 years of age, explain what happens to his rate of change in height. Use the graph of Height vs. Age to justify your answer.

8. Use your results from Question 5 to sketch a graph representing the approximate **rate of change in height** vs. the age for a boy at the 50th percentile from age 3 to 20. What happens to a boy's rate of change in height rate over time? Explain.



2 to 20 years: Girls
Stature-for-age and Weight-for-age percentiles

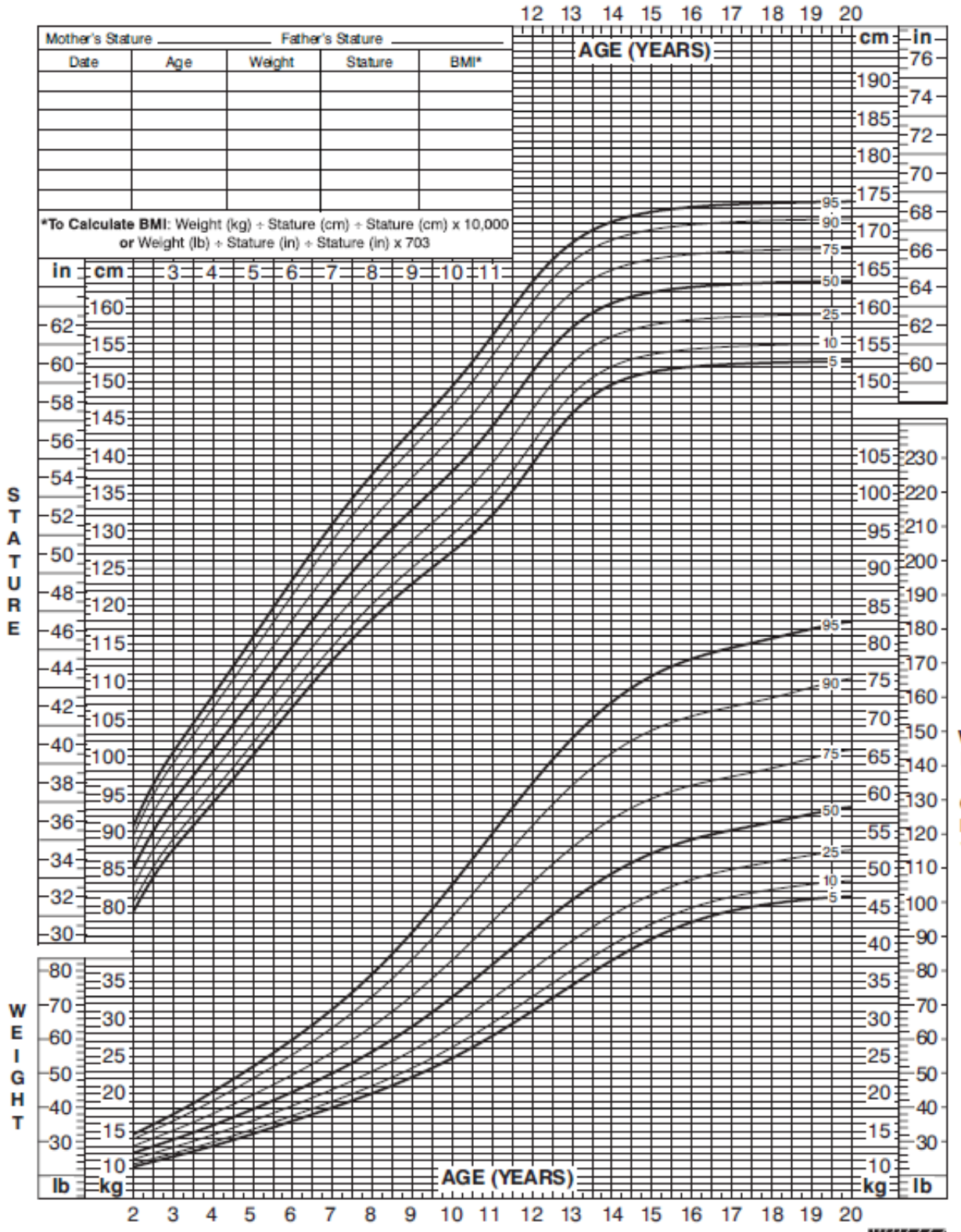
NAME _____

RECORD # _____

Mother's Stature _____ Father's Stature _____

Date	Age	Weight	Stature	BMI*

*To Calculate BMI: Weight (kg) ÷ Stature (cm) ÷ Stature (cm) x 10,000
or Weight (lb) ÷ Stature (in) ÷ Stature (in) x 703



Published May 30, 2000 (modified 11/21/00).
SOURCE: Developed by the National Center for Health Statistics in collaboration with the National Center for Chronic Disease Prevention and Health Promotion (2000).
http://www.cdc.gov/growthcharts

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2 to 20 years: Boys
Stature-for-age and Weight-for-age percentiles

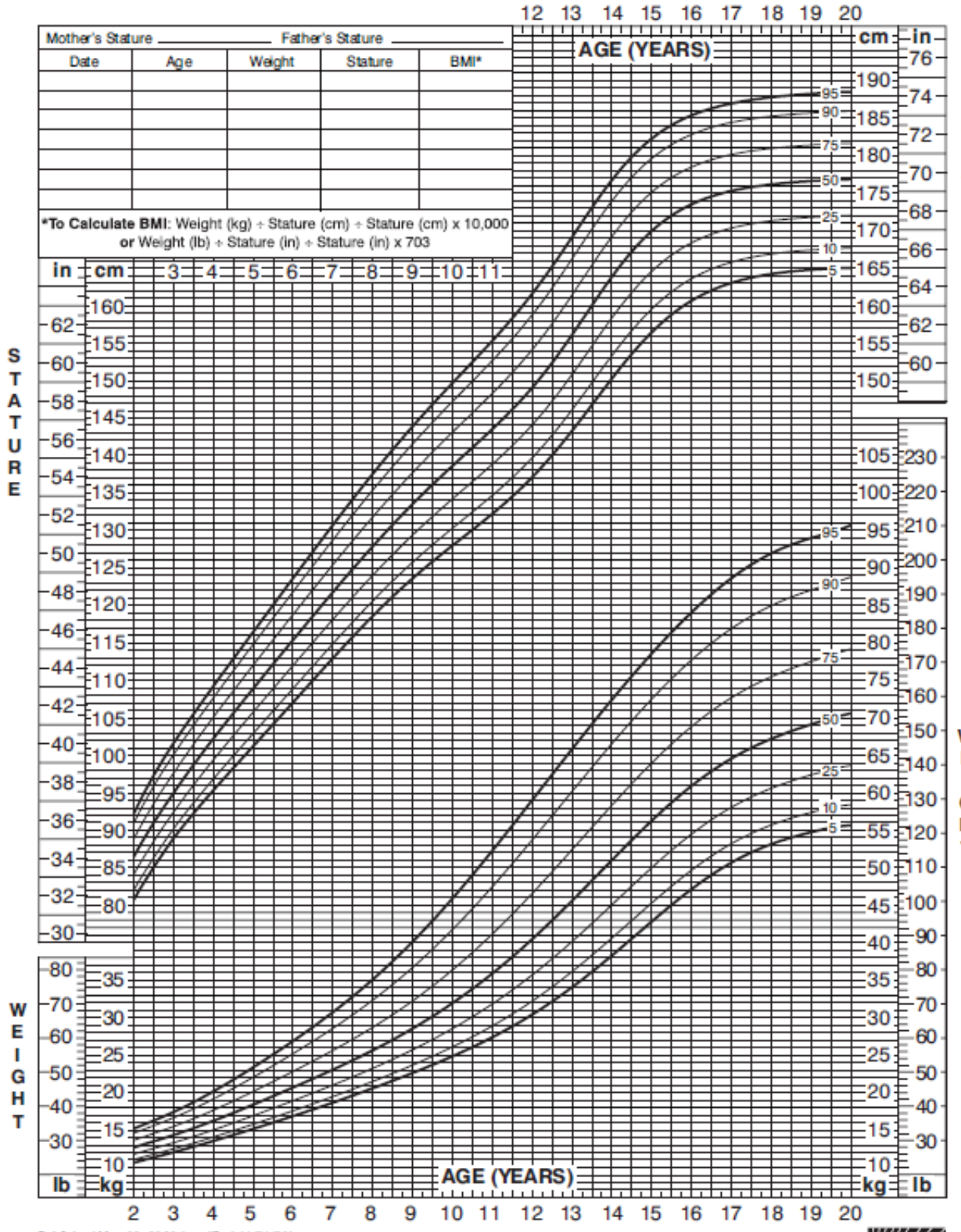
NAME _____

RECORD # _____

Mother's Stature _____ Father's Stature _____

Date	Age	Weight	Stature	BMI*

*To Calculate BMI: Weight (kg) ÷ Stature (cm) ÷ Stature (cm) x 10,000
or Weight (lb) ÷ Stature (in) ÷ Stature (in) x 703



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Even & Odd Functions

Algebraically: Even, Odd, Neither?

- ☆ A function is **even** if
  - ○ _____
- ☆ A function is **odd** if
  - ○ _____
- ☆ A function is **neither** if
  - ○ _____

What makes a term even or odd?

- ○ _____

Examples

Any number (-2, 65, ½)	x	x^2	x^3	x^4	x^5

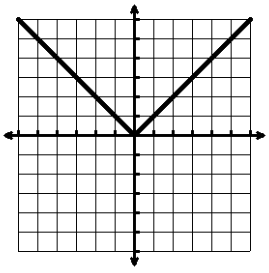
- ○ _____

Graphically: Even, Odd, Neither?

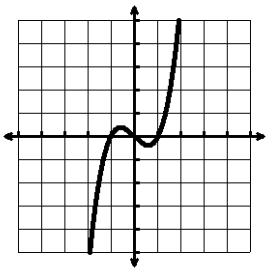
- ☆ A function is **even** if
  - ○ _____
- ☆ A function is **odd** if
  - ○ _____
- ☆ A function is **neither** if
  - ○ _____
  - ○ _____

Examples: Even, Odd, Neither?

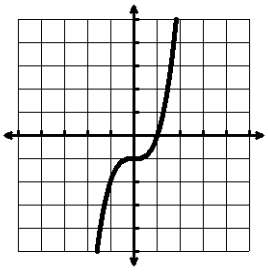
1.



2.



3.

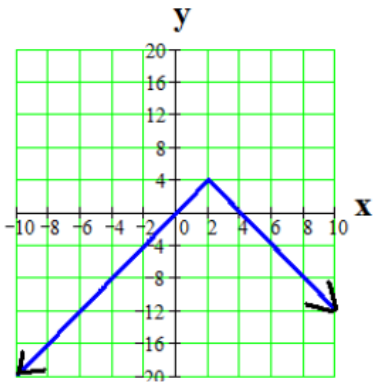


4. $f(x) = x^2 + 1$

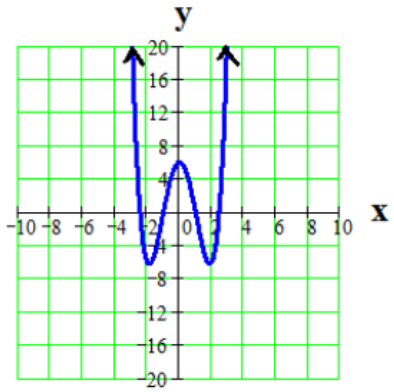
5. $f(x) = 2x^4 - 3x + 5$

6. $f(x) = x^3 + x$

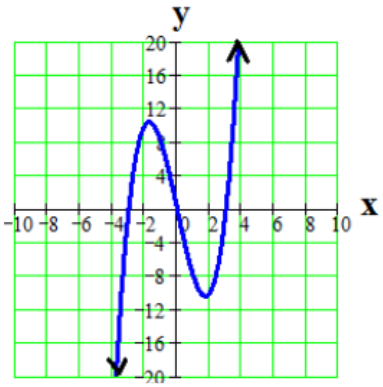
7.



8.



9.



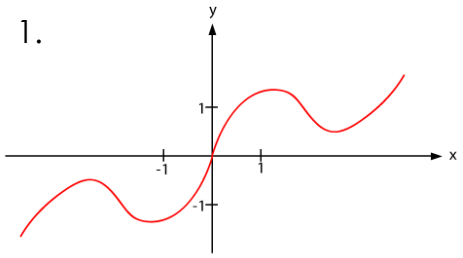
10. $f(x) = -3x^3 + 5$

11. $f(x) = x^2 + 4$

12. $f(x) = 4x^3 + 2x$

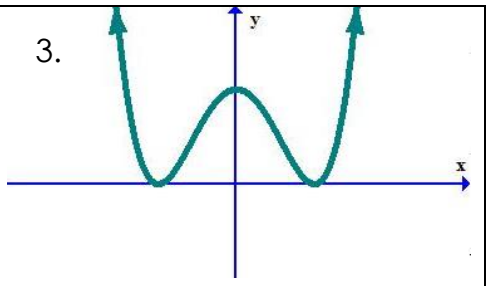
Even, Odd, or Neither?

1.



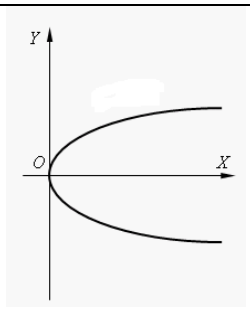
2. $f(x) = x^2 - 1$

3.



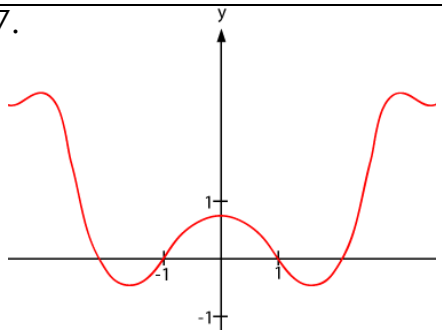
4. $f(x) = -5x^6$

5.



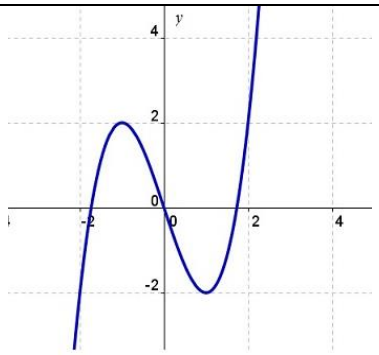
6. $f(x) = -2x^3 + 4x^2$

7.



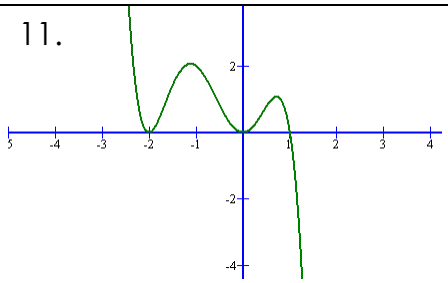
8. $f(x) = 9x^3 + 4$

9.

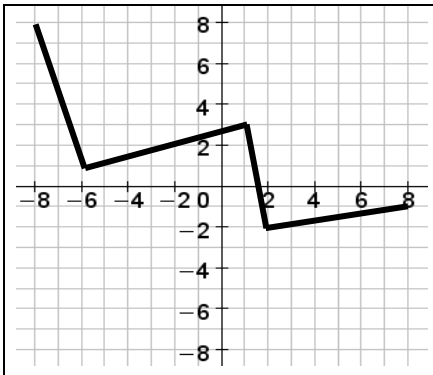


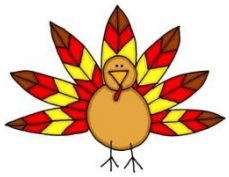
10. $f(x) - x^4 + 8$

11.



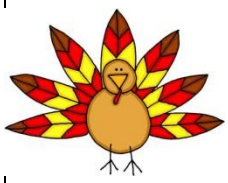
12. $f(x) = x^3 - x$





Use the graph to find the average rate of change from x = 1 to x = 2

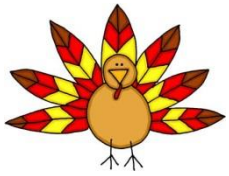
What is the average rate of change between (-3,0) and (4,2)



Use the graph to find the average rate of change from x = -6 to x = 1

Given f(x) = -3x – 8 , find the average rate of change when $x_1 = -2$ and $x_2 = 2$

Is this a function why or why not?
(12,1), (1,5) (5,6) (11,1)



Use the graph to find the average rate of change from x = -8 to x = -6

Given f(x) = -2x + 2, find the average rate of change when $x_1 = -4$ and $x_2 = 0$

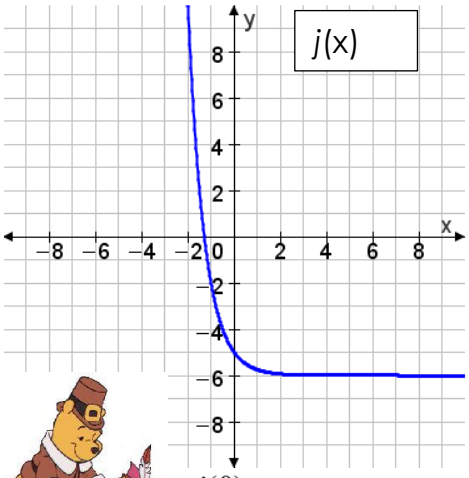
What is the average rate of change between (6,4) and (-2,-1)

Is this a function why or why not?
(0,1), (1,-3) (0,4) (-3,1)

Is this a function why or why not?
(12,1), (1,5) (5,6) (12,3)

Given f(x) = -5x – 4 , find the average rate of change when $x_1 = -1$ and $x_2 = 2$

What is the average rate of change between (0,-1) and (-2,5)



x	$h(x)$
-2	-6
-1	-4
0	-2
1	0
2	2

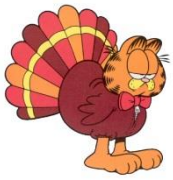
For $f(x)=(x-3)^2+1$, find the rate of change when $x_1 = 0$ and $x_2 = 3$

$j(0) =$ _____

$x =$ ____, *if* $j(x) = -2$

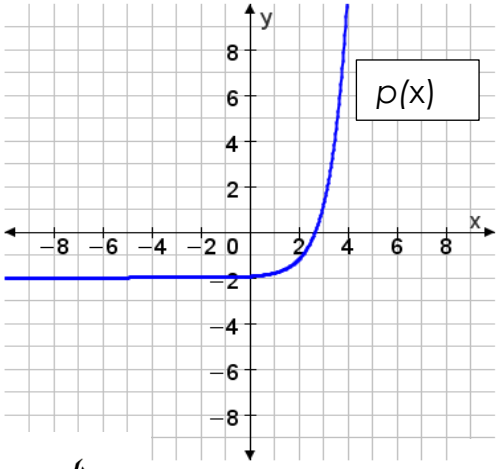
Given $f(x) = 3^x$, find the average rate of change when $x_1 = 0$ and $x_2 = 2$

$h(2) =$ _____



$x =$ ____, *if* $h(x) = -2$

For $f(x)=(x-1)^2-2$, find the rate of change when $x_1 = -2$ and $x_2 = 0$



x	$r(x)$
-2	4
-1	1
0	0
1	1
2	4

Given $f(x) = 5^x$, find the average rate of change when $x_1 = 0$ and $x_2 = 2$

$p(2) =$ _____

$x =$ ____, *if* $p(x) = 1$

$r(2) =$ _____

$x =$ ____, *if* $r(x) = -2$